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MATHEMATICAL MODELING AND VIBRATIONAL ANALYSIS OF HELICOPTER SLING-LOAD MECHANISM

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Abstract
Of the best machines that are employed worldwide for transporting or carrying heavy load over large distances, helicopter is the one. If we talk about the mechanism(s) used for the very job, there are the suspended cables below the helicopter. In response, disturbance forces such as wind coupled, with the helicopter motion the suspended load oscillates. The oscillations load carried by the suspended cables below the helicopter is not only dangerous but also affects the control of helicopter, and that too adversely. In the aerospace community the application of helicopter carrying external suspended loads is of significant focus owing to the stability problems in such systems. In the past, most of the work was focused on determining the favorable wire length, helicopter load ratios for stability analysis. In addition, the design of techniques that could augment stability in the slung load systems had also been the focus of past researches. In addition, a few of the researches focused on situation specific and simple models; however, most work in the past was focused on more general models that were suitable for control and simulation. The literature review, along with generic approaches to modeling slung load, discusses input shaping method to helicopter's flight controller in order to gain stability. So far as this paper is concerned, it includes mathematical modeling of the sling load in single degree of freedom. Additionally, in this paper we discusses different conditions to model sling load such as: un-damped free vibrations, damped free vibrations, vibration under harmonic forces and vibration under general forcing conditions. The finding of our paper presents the response of the sling load modeled in single degree of freedom under certain conditions. In this article we will learn as to how helicopter sling-load can be modeled and mathematically in single degree of freedom, and what is the response of the sling load under different conditions.

Keywords: Sling-Load; Mathematical Modeling; Single Degree of Freedom; Vibrations; Damped; Un-damped; Harmonic Excitation.

1. INTRODUCTION

We witness the employment of cranes for lifting heavy objects. Such cranes are designed for this purpose. However, when loads are to be carried over a large distance i.e. from tens of miles to hundreds of miles then helicopters do such job. So, keeping in view such a functionality, helicopters can rightly be termed as flying cranes. What makes helicopter a flying crane? It is the load suspended underneath aided by slings or cables. And, it is due to this very mechanism flying cranes or helicopters are very versatile; for it can transport almost anything i.e. from logs to power transmission towers; from rescuing people from remote areas to delivering food supplies for them. A few of the helicopter transport operations are shown in the figures 2,3, and 4 respectively. Now the question arises here as to what kind of helicopters do such jobs. One such example in this regards is the Kaman K-MAX R© Aerial Truck. Figure 5 is a manifestation of the utility of flying crane helicopters. If we talk about the utility of these flying crane helicopters, a couple of K-MAX helicopters have been employed for transporting over one million pounds of cargo to US Marine Corps deployed in Afghanistan over a span of 16 weeks.

Bisgaard et.al [1] presented a generic approach for modeling slung load systems that could aid the design engineers to handle not only wire slacking and tightening but also multi-lift systems. The model presented by Bisgaard et.al [1]gave an intuitive and easy-to-use approach to not only modeling but also simulating various slung-load suspension systems. It is not limited to but inclusive of response to, and detection of the tightening and slackening of wire. Moreover, the research also provides its readers with an intuitive approach to aerodynamic coupling between the slug-load and the helicopter.

2. MODESLING APPROACHES

There exist two distinct modeling approaches concerning the modeling of slung-load system under helicopters. The approaches include: the embedded formulation and the augmented formulation. The advantage of embedded formulation is that it formulates the problem in terms of degrees of freedom coupled with generalized. The advantage of such a formulation is that there are minimum set of equations; and moreover, constraint forces on the system are not revealed by such set of equations [2]. On the contrarys, the second type of formulation that is augmented formulation articulates the system in the form of redundant forces[3].

Glauert [4] was the first amongst all who considered the dynamics of the object carried by helicopter through sling, and he discovered all the problems that were involved in such a task. In addition, he identified that the two main causes leading to the instability of any such system included low load masses and short wires. His research was divided into three part wherein the first part dealt with wire system and subsequent determination of three periods of oscillations i.e. two in planar symmetry and third one being perpendicular to the first two. In the part two and three of his research the criterea for the longitudinal and lateral stability was developed respectively coupled with the introduction of two more periods of oscillations.

Moreover, Luscassen and Sterk [5] were the first ones who performed a thorough study on the helicopter slung load system. Although, the analysis performed by Luscassen and Sterk [5] were limited to three degrees of freedom; however, they were the first ones who considered the coupling between helicopter and slung load. The researchers in the following decades carried out research on the stability of slung load systems with the help of analytical models coupled with experimental testing of those analytical models. In a nutshell, almost all of the work was focused on determining a stable flight region vis-à-vis the parameters of slung load in order to avoid instabilities during the flight. A few of the researchers carried out analysis on certain parameters like the shape and geometry of the load to be carried in order to minimize the instabilities. Additionally, some other parameters were also considered that included fins, gyroscope, and drogues in order to reduce the instabilities in the system.

If we talk about the limitations in such model(s) devised by the early researchers, they were limited to or focused at stability analysis in the bounced region. In addition, a linearized model was also considered and studied that assumed no coupling between longitudinal and lateral motion [6]. Linearized model only dealt with forwarded flight and single-point suspension, and it also proposed that a long wire was needed for the stability. This model, by all means, proved to be beneficial for the later work that considered the utilization of gyroscope and fins to gain stability.

In addition, experimental testing using forced oscillations in a wind tunnel were also performed by Cicolani et.al [2]. And, it concluded that a two wire suspension system solved a few stability problem revealed in a single wire suspension system, and provided with an adequate suspension system with more stability. During the experimental testing in a wind tunnel, the flow filed and dynamics of the helicopter were ignored. Apart from the models that has been discussed above briefly, Newton-Euler equations can also be used to describe another suspension system for lifting heavy objects with helicopter. And, that specific system is inverted V-shaped suspension system Bisgaard et.al [1]. Moreover, this system also includes helicopter model based on stability derivatives. The basis of the model are inflexible and inextensible wires coupled with embedded system that reduces the constraining 12 degrees of freedom to 9 degrees of freedom. This system also incorporates the difficulties faced by engineers in order to obtaining an adequate model reaction to yaw-motion of the helicopter.

3. HELICOPTER MODELING

The design of flight control systems are assisted by helicopter dynamic models. Input shapers can be designed with the help of those models in which suspended load dynamics exist. This is done through the provision of estimates of the damping ratio and natural frequency of the load oscillation. The purpose of designing input shapers can be achieved substantially through these estimates. For obtaining measurements of the suspended load natural frequency, the need for reliance on a vision system which is a conventional method for designing input shapers – would be reduced. Usage of such models for designing input shapers will be investigated in this thesis. A simple model of the suspended load oscillation was verified by Potter et al. [21] and this model was put into use for testing how robust and effective different kinds of input shapers are and this was done by simulation. Conversely, control of the helicopter is affected by the suspended load oscillation in an adverse manner. This is due to coupling between the helicopter motion and the load swing. The model investigated by Potter et al. [21] did not incorporate this particular coupling effect into consideration. In this thesis, load-attitude coupling effects and load-back driving of the helicopter will be included in the models which are to be investigated. The design and subsequent testing of helicopter flight control systems for operations of suspended load can be done through relatively more sophisticated models that take into account more of the helicopter dynamics. In this thesis, such a sophisticated model will be employed to examine the combination input shaping with a flight control system for usage during suspended load operations.

3.1 Dynamic Modeling of Helicopter

James [11]addressed the dynamic modeling of helicopter carrying suspended load. The main goal of the dynamic modeling was to come up with such a model that yields estimated damping ratio coupled with natural frequency of the suspended load swing for load configurations and range of helicopters[25]. Dynamic modeling of the helicopter carrying suspended load was an effort to producing sufficiently accurate damping ratio and natural frequency estimates that could be employed in designing input shapers. Moreover, the dynamic modeling helped to identify the dynamic effects that are important to helicopter-suspended load operations. Following the design of dynamic model, it was employed for designing and simulation testing of flight controllers that were proposed for suspended-load operations.

Several researchers carried out a research on the design on the design oy dynamic modeling of helicopter carrying suspended load. Dukes [24] studied helicopters carrying suspended loads using simple models consisting of only a few degrees of freedom. The models studied by Dukes [24] were used to analyze such methods that could damp the pendulum mode of the swing load for different maneuvers performed by the helicopters. Moreover, Potter et al. [21] embarked on translational model, and his motivation towards studying such a model was to approximate the suspended load dynamics by studying the similarities between cranes and helicopters carrying suspended load

Lucassen and Sterk [5] jumped on the same bandwagon and proposed in their work that a helicopter carrying a suspended load near hover is similar to the double pendulum, albeit mechanically. Additionally, Bisgaard et al. [1] contributed to the cause in such a manner that he modeled distinct slug load suspension types that included multi-lift configurations. The results were also verified experimentally, albeit on a small scale helicopter.

It has also been established that the dynamics of the crane payloads and the loads suspended from a helicopter are similar to each other [11]. In fact, the dynamic modeling of both of the systems can be as simple as the dynamic modeling of a pendulum. So, it was justified that the modeling of helicopter load suspension system which is quite complex could be simplified by employing same techniques that were used to model cranes coupled with the complex dynamics that includes load-vehicle coupling [26]. Simple dynamic models of the helicopter load suspension system was derived by relaxing a few assumptions that were employed in deriving the dynamic models of cranes.

Potter et al. [21] embarked on a similar approach i.e. coupling a first order model with a simple but second-order underdamped model of the swing or payload swing. This coupling helped to approximating transnational dynamics with helicopter attitude. James [11]presented a planar model where suspending payload was allowed to back-drive the suspension point. This back-drive effect is common in heavy sling loads.

Cao et.al [28] presented a dynamic model of helicopter slung-load system under the flexible sling hypothesis. The model was comprehensive base and physics based that was established via Kane's method[29]. The first model of the dynamic model for flexible sling-external laod system was developed on the basis of spring-point mass-damper model. The slung-load system of the helicopter with flexible sling can be seen in Fig. 1.

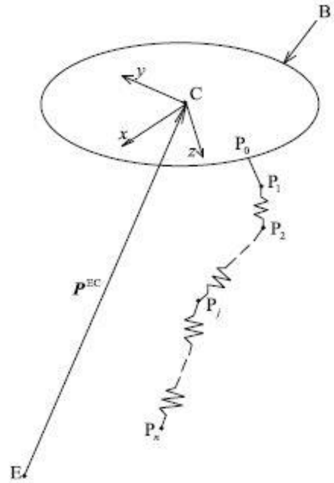

Fig. 1 The slung-load system of the helicopter with flexible sling.

Rigid body "B" is the manifestation of helicopter, and "C" illustrates the mass center. Point P0 is the hook point on the helicopter. As far as the modeling of sling is concerned, it is modeled as n-1 mass points i.e. P1, P2,….., Pn-1 where (n>1). These points are connected via spring chains. The gravity force was considered concerning the flexible sling; however, the aerodynamic force was neglected. As the aerodynamic force was neglected, the sling was divided into "n" parts and the portion in the figure i.e. P0, P1 is a rigid pole. As far as the remaining parts are concerned, they were taken as spring dampers. The aerodynamic force and gravity of the slung-load was considered and it was simplified as point mass Pn. Point "E" was taken as fixed reference point the ground. If we talk about helicopter, x-axis, y-axis, and z-axis in the figure above are the body axis system of helicopter and are taken in positive direction. The unit vectors b1, b2, and b3 were defined for the x, y, and z- axis respectively. The formula given was: b3= b1 x b2. A space vector was introduced to define the hook point. Lastly, forces on the hook point that represents the pull of the sling were also defined.

In another research carried out by Cao and Wang [30] another dynamic model of helicopter and slung load was presented which was based on rigid body rigid body slung-load hypothesis. The model presented by Cao and Wang [30] included 19 DOF that incorporated rigid body slung-load and helicopter full motions. A thorough analysis of helicopter stability and with rigid body slung-load was carried out based on the trimmed results. The trimmed results were established following the calculation of trimmed values of state variables at distinct velocities; and the method that was used for this, was continuation method. The paper established that the introduction of slung-load not only exert external forces but also effects the dynamic stabilities of the helicopter.

4. MATHEMATICAL MODELING

In this section, helicopter sling load will be modeled in different conditions in single degree of freedom and two degrees of freedom. As far as the free body diagram of concerned, it is manifested in the picture below. However, prior to discussing different conditions for mathematical modeling, it is necessary to define certain parameters that will be used to study the response of the system are listed in the table below:

Table 1. List of Parameters Used in Mathematical Modeling.

PARAMETR	FORMULA	VALUE
CABLE MASS	N/A	80 Kg
MASS ATTACHED	N/A	2000Kg
EQUIVALENT MASS:	$m_{eq.} = M + (33/140)(m)$	2019 Kg
CABLE DIAMETER	N/A	0.038 m
CABLE LENGTH	N/A	10 m
CROSS-SECTION AREA	$\pi/4\ (D^2)$	1.134×10^{-3} m^2
YOUNG'S MODULUS	N/A	2.1×10^{11} N/m^2
CABLE STIFFNESS	k= EA/L	2.3814×10^{7} N/m
NATURAL FREQUENCY	$\sqrt{k/m_{eq}}$	108.6 rad/sec
DAMPING RATIO	C/C_c	0.5
DAMPING FREQUENCY	$w_d = \sqrt{1-\zeta^2}\, w_n$	94.05 rad/sec
EXCITATION FREQUENCY	N/A	70 rad/sec
FREQUENCY RATIO	$r = w/w_n$	0.64
FORCE	N/A	2000 N
STATIC DEFLECTION	F_o / k	8.399×10^{-5} m

In the following section there will be a discussion about as to how sling load mechanism can be modeled mathematically in different conditions in single DOF i.e. Un-damped free vibrations, damped free vibrations, Un-damped vibrations under harmonic force, damped vibrations under harmonic force, and vibrations under general forcing conditions (particularly constant force). So, we will discuss all these conditions and their response equations one by one.

4.1 **Un-damped Free Vibrations** In this condition, it is assumed that system that consists of sling load is vibrating freely. Free body diagram of the system is shown in figure 2.

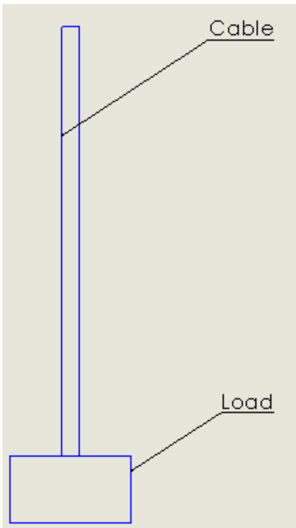

Fig.2 Free Body Diagram of Helicopter sling load in single degree of freedom

The response equation of the system is given by:

$$x\,(t) = X_o \cos(w_n + \emptyset) \qquad (1)$$

Whereas the values of Xo and ɵ for the un-damped free vibrations are as follows:

$$X_o = [\,x_\circ + (\frac{x_o}{w\,n})^2\,]^{1/2} \qquad (1.1)$$

$$\emptyset_\circ = \tan^{-1}[\frac{x^\circ{}_o\,w\,n}{x_o}] \qquad (1.2)$$

As far as the initial conditions are concerned, they are taken as follows:

$$Displacemen\ (x_\circ) = 0.25 \text{ m at (t=0)} \qquad (E1)$$

$$Velocity\ (x^\circ{}_o) = 0\,\frac{m}{s} \text{ (t=0)} \qquad (E2)$$

4.2 Damped Free Vibrations In this conditions, a damper will be introduced in the system and the damping ratio ʆ is taken as 0.5 that means the damping co-efficient "C" of the damper is on half of the value of critical damping "Cc ", as it can also be seen in the table 1 above. The free-body diagram of the system is given in figure 12 below:

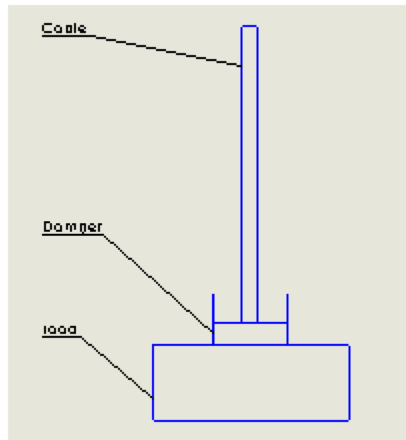

Fig.3 Free Body Diagram of Helicopter sling load with damper in single degree of freedom

The response of the system can be defined by the following set of equations:

$$x\,(t) = X_o\,e^{-\zeta w\,nt}\sin(w_d\,t + \emptyset) \qquad (2)$$

$$X_o = [x_\circ^2 w_n^2 + (\mathring{x_\circ})^2 + 2\,\zeta x_\circ \mathring{x_\circ}\,]^{1/2} / w_d \qquad (2.1)$$

$$\emptyset_\circ = \tan^{-1}\frac{x_0\ \ w\,d}{\mathring{x_\circ} + \zeta w\,n x_\circ} \qquad (2.2)$$

4.3 System Under Harmonically Excited Damped Vibrations In this condition, the system response will be modeled under harmonic excitations through the following set of equations:

$$F(t) = F_0 \text{ Cos wt} \qquad (A)$$

In the above equation "w" represented excitation frequency and its values can be taken as 70 rad/sec as mentioned in Table 1 above. So far as free-body diagram is concerned, it is same as shown in the figure 12 above. The only difference is that harmonic force is acting on the system as manifested in the (A). The response of the system is given by the equation written below:

$$x(t) = X_\circ e^{-\zeta w_n t}\cos(w_d t - \emptyset_\circ) + X\cos(wt - \emptyset) \quad (3)$$

$$X_\circ = [x_\circ^2 w_n^2 + (\dot{x}_\circ)^2 + 2\zeta x_\circ \dot{x}_\circ]^{1/2} / w_d \quad (3.1)$$

$$\emptyset_\circ = \tan^{-1}\frac{x_0\, w_d}{\dot{x}_\circ + \zeta w_n x_\circ} \quad (3.2)$$

$$X = \delta_{st} \times \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad (3.3)$$

$$\emptyset = \frac{2\delta r}{(1-r^2)} \quad (3.4)$$

4.4 Vibrations Under General Forcing Conditions In this condition the system will be modeled in such a way that constant force of 2000N is acting on the system. The free-body diagram of the system is same as manifested in figure 12. Moreover, the initial conditions will remain same as manifested in (E1) and (E2) above. The response of the system under constant force (Fo = 2000N) can be given by the equations given below:

$$(t) = \delta_{st}\ [1 - \frac{1}{\sqrt{(1-\zeta^2)}} e^{-\zeta w_n t}\cos(w_d t - \emptyset_\circ)] \quad (4)$$

$$\emptyset = \tan^{-1}\frac{\zeta}{\sqrt{(1-\zeta^2)}} \quad (4.1)$$

5. IMPLEMENTATION

In order for the above response equation to be solved or implemented, MATLAB serves the purpose quite efficiently. If we talk about the pros and cons of employing MATLAB for solving response of the system through the equations discussed in previous section then they are as follows:

5.1 **Pros** The pros of using MATLAB for calcylating the system response are as follows:

I. It has a huge built-in library for functions and tools, so it is simple to utilize Matlab when used properly.

II. Due to the built in functions of ODEs it was easy to use MATLAB to solving for the system response.

III. One of the best advantages of using MATLAB is that it uses intuitive approach and helps the user a lot when it comes to matrix manipulation.

IV. As far as plotting and graphing is concerned, it is quite simple in MATLAB for unlike excel, one needs not change the scale from normal to logarithmic which in some cases is difficult when the output value is negative.

5.2 **Cons** The cons are listed as follows:

I. One cannot use MATLAB sans prior practice.

II. Unlike MS Excel, one must learn coding technique of MATLAB in order to solve the problems.

III. The picture quality of the graphs is not as good as in case of other plotting programs.

IV. Sometimes it is hard to locate syntax error, if any, which prove to be cumbersome for the user.

6. RESULTS

In this section, results are discussed one by one in the form of graphs that we obtained following the solution of the response of our system in the conditions mentioned above.

6.1 Un-damped Free Vibrations Un-damped Free vibrations are oscillations where the total energy stays the same over time. This means that the amplitude of the vibration stays the same. Un-damped free vibration happens at the natural frequency of the body. This natural frequency plays an important role in damage mechanics. It is called “free” because there is no external energy source driving the movement (after the sudden blow that starts the movement).

As far as our case is concerned, the response of system concerning un-damped free vibrations was solved through MATLAB. The response is shown in the figure 4 below:

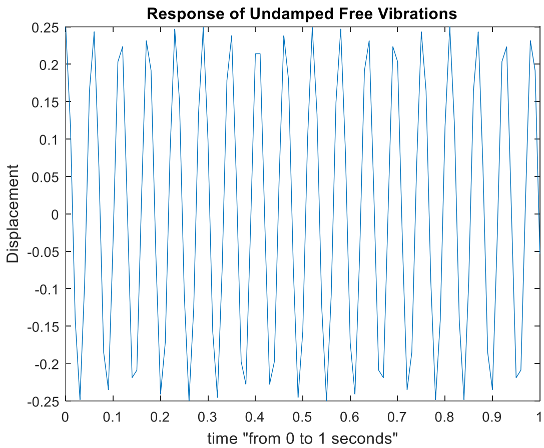

Fig.4 Response of Un-damped Free Vibrations

The plot in the above picture shows that the system will keep on oscillating with natural frequency unless and until resistive force i.e. damping is applied on it. Moreover, free vibrations are practically not possible; and the reason is that such systems neither lose energy nor gain energy during this process. Vacuum can only render free vibrations of sound, and such vibrations are said to be ideal. Practically, it is impossible to apprehend the phenomenon of free vibrations. However, the above picture only shows that how the system will behave vis-à-vis un-damped free vibrations.

6.2 Damped Free Vibrations When a body vibrates with its natural frequency and the amplitude decays with time and finally the body comes to rest at its mean position. Such vibration is called damped vibration. So far as the damping the system is concerned, the system is underdamped i.e. the damping "C" is less than "Cc". Now, the question arises here that why underdamped systems are desirable. Underdamped systems are the most practical and most commonly used. An underdamped system ensure the system always reaches the desired end state with some overshoot. Even though there is overshoot the damping eventually brings the system to the desired state. The response of the system for damped free vibration is shown below in the figure 5

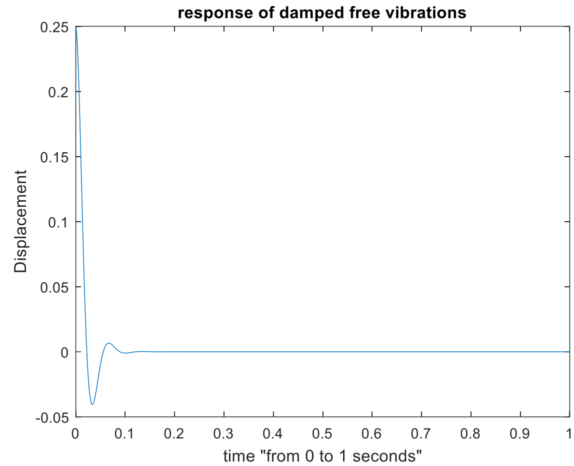

Fig. 5 Response of Damped Free Vibrations

As it is evident from the above graph that system comes to normal state after a very short period of time i.e. 0.1 seconds. This is because the higher value of stiffness and corresponding high value of critical damping. As far as the damping is concerned, it is one half of the critical damping.

6.3 Damped System under Harmonically Excited Vibrations in this condition, the system under harmonically excited vibrations will be discussed. As far as the solution of such systems are concerned, they are of the form x(t) = xh (t) + xp (t) i.e. the solution consists of homogenous part and the particular part. The homogenous solution xh (t) is due to the natural frequency of the system whereas, xp (t) occurs due to the excitation frequency or forced frequency. Due to damping the oscillations due to the natural frequency will die out as discussed in previous section (Figure 6), and the system will come to it steady state i.e. the oscillations due to the excitation frequency or forced frequency. However, it must be taken care of that the ratio of forced frequency to natural frequency i.e. r = w/w n is never equal to 1. Because, in such a case the system will experience resonance and could fail or damage catastrophically. So far as the response of such a system is concerned, it as shown in the figure 6 below.

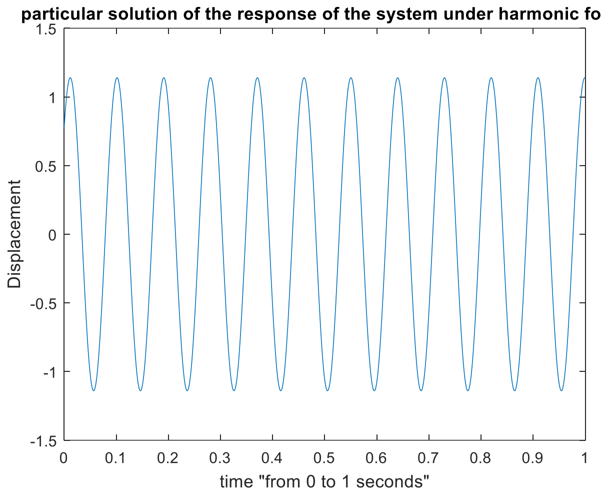

Fig. 6 Particular Solution of the Response of the System Under harmonic Force

The response only captures the particular solution because due to damping the oscillations due to natural frequency will die out. The above figure manifests the steady state response i.e. after the natural is vanished, this is how the system behaves under forced frequency which, of course, is less the natural frequency.

6.4 Vibrations Under General Forcing Conditions (Constant Force)

In this condition, the response of the system under the step input or constant force is discussed. As we can see in the figure 7 that system response dies out after 0.1 seconds. This is due to the introduction of damper in the system. As we know that the degree of damping indicates the nature of transient system. There will be no damping at all if the ratio is equal to “Zero”, and the system will continue to oscillate indefinitely. The ratio when increased from 0 to 1 (0 to 100%), will reduce the oscillations, with exactly no oscillations and best response at damping ratio equal to 1. On further increasing the damping ratio, the degree of damping has been overdone, this will cause sluggish performance/longer transients in the system.

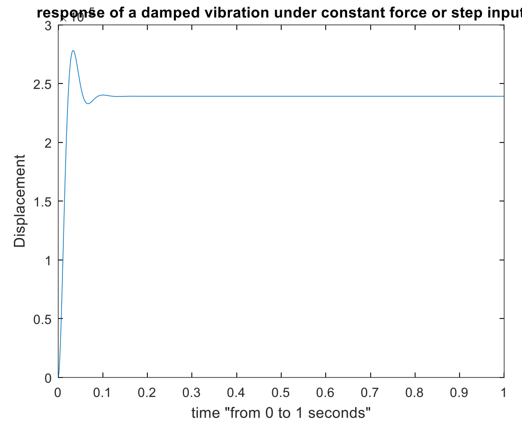

Fig. 7 Response of a Damped Vibrations under Constant Force or Step Input

7 CONCLUSION

The response of the system under different conditions was studied. Only the free vibrations were discussed without damping and, all the other cases were studied and discussed with damper. It is because the dampers are inevitable in the vibrating systems. Otherwise, the oscillation with natural frequency will damage the system. Moreover, the oscillations die out very rapidly in our case because the stiffness is very high and corresponding value of the critical damping is also very high. So, it was inevitable to use with damping ratio of at least half. Damping ratio describes how rapidly the Amplitude of a Vibrating system decays with respect to time. If damping ratio is less than 1 then the system is called Under Damped. Vibratory system is critically damped if the ratio of the damping equates 1 and system is called over damped if the ratio of damping is greater than 1. Under Damped system, when excited by a force, oscillates and comes to rest gradually with decaying amplitude. Over Damped system doesn't vibrate at all. In case of Critical Damping the system gets displaced from its equilibrium position and in return the system does not overshoot the equilibrium position immediately and comes to rest in a very short interval of time. Mostly the systems are underdamped because the ratio of damping for such system will always be less than 1.

8 RECOMMENDATIONS

In order for system to be modeled more accurately, it is recommended that system be modeled in two degree of freedom system. Two degrees of freedom system will further refine the response of the system in different conditions. Moreover, it is also recommended that vibration produced the helicopter movement, its rotor RPMs etc. must also be considered for better understanding of the system. In this system, all the responses were studied for only material; so, in future it is recommended that at least two materials of the sling cable be tested in different condition with at least two sets of the parameters shown in the Table 1. If we talk about the implementation of the mathematical model of the system discussed in this paper, it is good for understanding the fundamentals of the oscillations in the helicopter sling cable. However, there is a room for further refinement in terms of mathematical models for understanding and studying the response of the system.

NOMENCLATURE

M	Mass attached to the string
D	Cable Diameter
L	Cable Length
A	Cross-section Area
E	Young's Modulus
W_d	Damping Frequency
F_o	Force
m	Cable mass
$m_{eq.}$	Equivalent Mass
k	Cable Stiffness
w_n	Natural Frequency
w	Excitation Frequency
r	Frequency Ratio
ζ	Damping Ratio
δ_{st}	Static Deflection

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