AUJoGR-22404

MODELING, DESIGN AND ANALYSIS OF AUTOMOTIVE ENGINE PISTON LUBRICATION AND DAMPING SYSTEM

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Abstract
In internal combustion engines the piston reciprocates due to the combustion forces. It reciprocates having a clearance between cylinder liner and the piston skirt and this clearance causes the piston to move laterally as well. In this work a mathematical model have been developed to find the response of reciprocating motion as well as the lateral or secondary motion with total of two degrees of freedom. For this purpose the combustion force have been transformed using Fourier transform method. The forces acting in both direction such as primary direction and secondary direction (lateral) have been plotted along with the response of the piston under the influence of these forces. For this purpose a 127 cc motorcycle piston have been considered, while modeling the damping the viscous properties of SAE 30 motor oil have been used, also the motor oil lubrication is assumed hydrodynamic and also as a Newtonian fluid. Similarly the stiffness of the system have been derived using rotational work energy principle. The equations have been solved and plotted using Matlab.

Keywords: secondary motion; piston; vibration; Fourier transform; response; Mathematical Modeling; two degrees of freedom; damping

1. INTRODUCTION

Piston converts the energy from combustion and expanding gases to reciprocator motion. But there are some energy losses due to frictions and the frictional losses in piston accounts for almost 50% and in some cases 30 – 45%. The piston has two types of motion while it works such as linear reciprocator motion and secondary motion. The linear reciprocator motion is essential but the secondary motion which is mainly the lateral motion of the piston because of the clearance between the piston skirts or in some cases piston rings and the cylinder liner. This clearance also allows the piston to rotate around the piston wrist pin. Hence there are two types of secondary motions which cannot be avoided because the piston to operate there must be some amount of clearance between the cylinder liner and the piston skirt and the clearance ultimately allows the secondary motion.

The piston is contained in the sleeve and the rings of the piston which are assembled to the piston at its respective grooves helps in the sealing of the combustion chamber. Basically the three types of rings are termed as ring pack, the top ring is called compressing ring the second scraper ring and the third is called oil control ring, these rings as well as the piston skirt are connected to the cylinder liner by hydrodynamic contact.

And for the purpose of the reduction of the friction between the two mating surfaces the engine oil is used as lubricant. Piston is a part which experiences the pressure of the expanding gases and compressing gases and these forces either makes the piston move and also produces some stresses in the piston. The main concern in case of the piston is that the processes in the problems of piston repeat at varying frequencies and so is the secondary motion of the piston. Even if the surfaces are lubricated the secondary motion allow the surfaces to come in close contact and even Elast-hyro-dynamically which give rise to the noise, friction and even elastic deformation of the mating surfaces.

When solving problems like this it needs the knowledge of fluid mechanics, theory of elasticity, vibrations and tribology. First the model of the lubrication is decided first and then the coupling of piston lubrication and the cylinder liner are modeled. The Literature review suggest that the partial elasto-hydrodynamic lubrication model is more realistic approach in solving Tribological problems because it includes the wear phenomena, the elastic deformation of the mating surfaces and the hydrodynamics principles are still not violated and there is a constant flow into the film of oil maintained by the lubrication system.

There is an active lubrication system working in the automotive engine which serves to lubricate the crank shaft, piston and every mating surface and this lubrication is made possible without the influence of external pressure and a hydrodynamic lubrication is thus made possible by continues supply of the lubricating oil into the mating surface to maintain film of oil. The pressure that is acting on the surfaces is due to the fluid shear and the pressure due to hydrodynamics of the film.

2. LITERATURE REVIEW

There have been large sum of research carried out on the tribology of the piston and cylinder and the lubrication problem has been modeled with Hydrodynamic Lubrication (HL), Elasto-Hydrodynamic Lubrication (EHL), Partial Elasto-Hydrodynamic Lubrication (PEHL) and Mixed Lubrication (ML) The engine oil has many other functions as well but in our peace of work three functions of the lubrication engine oil have been considered such as lubrication, cooling of the mating surfaces by convection and damping of the secondary motions. In these functions according to the studied died literature the lubrication and the Tribological performance have been thoroughly studied but the damping of the secondary motion by the engine oil still need attention and that is why it has been considered. The torsional vibration analysis were carried out for Multibody single cylinder internal combustion engine model comprising inertial rigid bodies, bearings for support, damping components , connections and couplers between some of the engine components. Using this model[1] some of engine designs prototypes can be tested. This model also investigates noise and vibration. The new approach numerical analysis of dynamics problems which are Multibody and complex nonlinear has been brought up. Fig. 3 shows the combustion force behavior over time

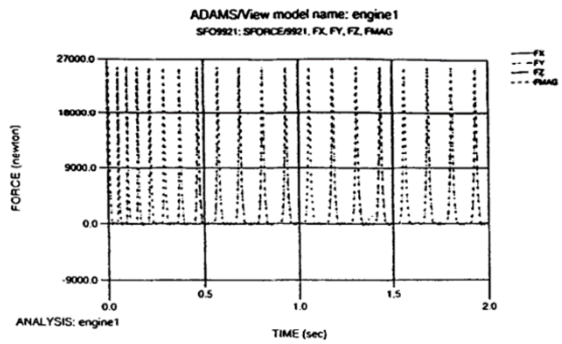


Fig. 3. Combustion Force behavoiur over time [1]

temperatures of engine oils has measured for two types of oil such as new oil and used oil of a passenger car and in this work Citroen Berlingo car[3]. The viscosity of oil selected is 5w30. Engine oil is used both in internal combustion engines of both diesel and petrol or gasoline fueled. And viscosity is the major property of lubricating oil as it is the internal friction of moving oil particles, kinematic viscosity decreases with increasing temperature[3]. The dynamics and lubrication of piston are analyzed using a developed method in finite Element Method. First numerical modal has been developed for Multibody dynamics including for piston connecting rod and crack. LaGrange multipliers [4] and constraint Jacobean matrix has been created for solving the Multibody dynamics equations. And the dynamics and lubrication model have been coupled[4]. They also investigated that how some of the piston design parameters affect the noise from slap and the lubrication performance. Some parameters including bore clearance, curvature parameter of skirt profile and bulge position. The results suggest that if the clearance is small than the smaller would be piston secondary motion due to less space, and hence it results in high slap noise and wear. Also it produces higher oil film shear stress. And the piston skirt position of barrel peak have effect on the

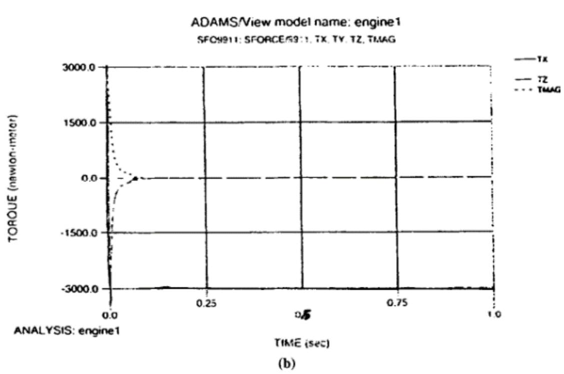


Fig. 4. Torque of starter motor to start the engine [1]

Fig. 4 shows the torque of starter motor to start the engine and process of reciprocation. Harmonic analysis using commercial engineering software Ansys has been done in three dimensions as well as in two dimensions for Aluminum material such as HF 18 for different compression pressures of 40-75[2] bar of range in Variable Compression Ratio (VCR) cylinder liner of diesel engine

Using FUNGILAB SMART series V210003 rotary viscometer[3] the kinematic viscosity over different characteristics of piston skirt-liner lubrication and dynamics[4].

The lubrication performance of piston skirt has also been worked out by [5] through some design parameters. The computer code used for this analysis is called Friction and Lubrication Analysis of reciprocating Engines (FLARE) .The type of lubrication selected is Mixed Elasto-hydrodynamic lubrication [5]. And thermal and elastic deformations have been considered for skirt also for the fine model the effect of piston tilt and piston load and variation in speed also been considered. The lubricant pressure has been calculated using the Reynolds equation. The film thickness is decided by the magnitude of clearance between the piston and the cylinder liner. As the model developed include three type of piston and cylinder liner interaction such as in which there is a lubricant film between the surfaces and other solid to solid hydrodynamic contact and one with no contact and a cavity is present between the two surfaces. For those regions where the solid to solid contact comes into play the pressure has been find out by using elasticity equations. And the combined effect has been find out by using Murty's method. Further where Reynolds's number has to be solved Newton-Raphson (NR) method has been used. And in each step of NR method the pressure of the oil film, pressure from contact and secondary motion of the piston are solved simultaneously.

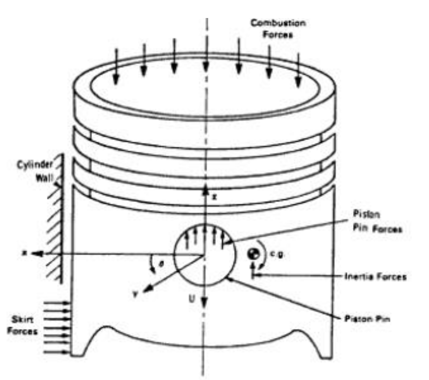


Fig. 5. Applied and reaction forces on the piston [5]

The Fig. 5 shows the applied and reaction forces in the model such as the combustion forces and its reaction at piston pin, the restraints by cylinder wall, skirt forces, inertial forces in the piston and moments along y direction which tends to tilts the piston [5].

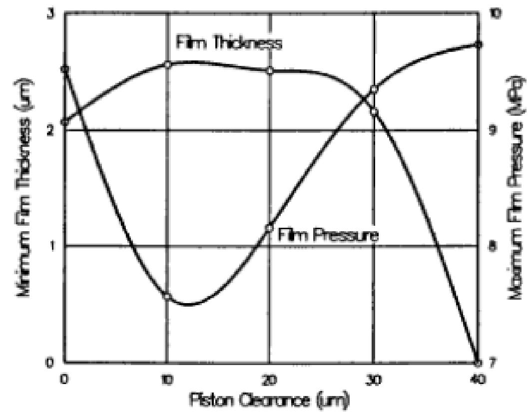


Fig. 6. Film thickness, Pressure vs piston clearance [5]

Fig. 6 shows film thickness first increases with clearance to 10 um[5] and then start decreasing abruptly after 25 um and the pressure decreases at first and then start increasing with increasing the clearance. The force of viscous damping of MR damper prototype increases slowly[6] as compare to bobbin-in-piston configuration and thus results in great dynamic range. It has the ability of greater stroke while in hydraulic cylinder. Using solid works a CAD model is developed for an internal combustion engine reciprocating piston and at the speed range of 600 rpm to 3000 rpm[7] the model is simulated for certain results like the reaction forces, linear velocity against the different times of combustion, thermal stresses and equivalent strains in the piston and the piston displacements are being determined. The geometrical features of the piston is shown in Fig. 7.

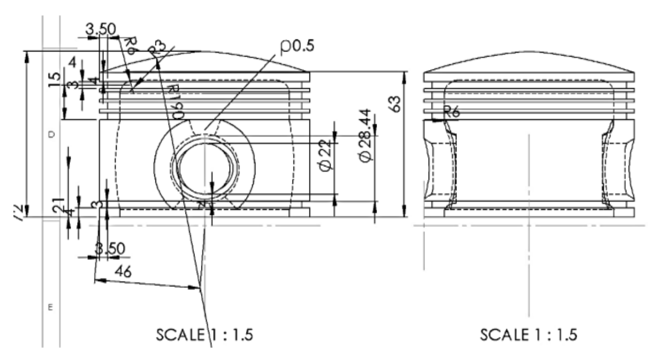
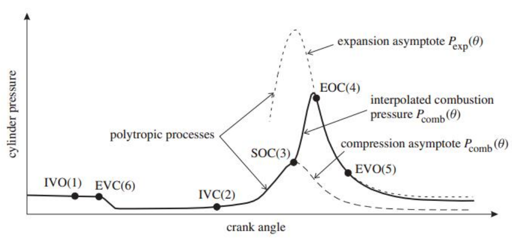


Fig. 7. Piston Dimentions [7]

Importance of textured surface of Tribological surfaces has been discussed by [8] as the textured piston ring reduce the friction by 20 percent to 50 percent as compare to the un-textured [8] ring and decreases 4 percent of the total fuel consumption. In another work [9] Finite Element structural and thermal analysis has done using five different types of materials such as Al-sic Graphite, AL-GHY 1250, A7075, A4032, and A6082[9]. The analysis is done using Ansys 15 static and a piston from four stokes and single cylinder engine has been selected. The results for total von misses stresses, Equivalent Strains and Total deformation have been obtained for all of the five materials. On the basis of these results Al-sic graphite has been suggested to be used for piston material. [10] using the gas pressures with in the cylinder the piston motion has been determine of the internal combustion engine and not just assuming the uniform constant rotation of the crank, for the secondary motions the damping have been determine in lubrication condition of fully flooded. The variations in pressures can be modeled by Mass Fraction Burnt (MBF) [10] profile of the mixture of both air and fuel. The MBF nonlinear interpolation in the two pressures traces such as asymptotic pressure traces P_{comp} and asymptotic pressure traces P_{exp} which is illustrated theoretically as in the following figure. Also the pressure has been measured and that calculated are also shown in Fig. 8.



Cylinder pressure model based on non-linear MFB interpolation between expansion and compression asymptotes [**20**]

Fig. 8. Non Linear MFB interpolation model for cylinder pressure [10]

In 1982 Steve M. Rohde, Dennis F. LI and Hazem A. Ezzat in general motors research laboratories Michigan conducted a classical analytical study on piston dynamics in a reciprocating engine. [11] studied that how the clearance and other properties and parameters changes the piston secondary motion such as piston secondary translation and rotation on piston pin axis, and also how the piston skirt frictional power loss[11] behave. Other parameters have also been analyzed for the effect on the secondary motion and frictional loss like the lubricant viscosities and more significantly the piston pin location. The approach that has been developed by [11] can be used to select the parameters for designing the engine and operating conditions. And the piston slap effect on the vibration and noise of engine can be assessed easily[11][12]. have developed a new mathematical model for the dynamics of piston in the mathematical modal the connecting rod and crack shaft dynamics are also been considered which are mainly ignored in many models for analyzing the secondary motions of the piston.

Cristiana Deplete, Abbas Razavykia and Paolo Baldissera [13] developed an analytical model to study the contact behaviors of piston skirt and the cylinder liner. They have selected the hydrodynamic lubrication and packed lubrication using piston ring. And for determining the friction forces for the piston ring pack and its respective moment in the axis of wrist pin the nonlinear equations system have been solved and the governing equations is Reynolds equation and Force equilibrium. Three types of boundary conditions have been used to determine the change thickness of minimum film of oil at the interface of piston ring and liner [13].

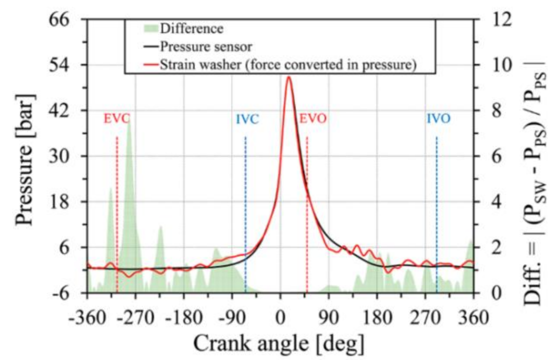


Fig. 9. variation of pressure in cylinder against Crank angle [14]

The in-cylinder pressure has been measured using piezoelectric sensor and that measured using the strain washer are shown in Fig. 9 which are in close agreement with relative error at certain crank angle. And further the method has also been applied to conditions such as misfiring, knocking and pre-ignition. And the method developed can be used for multiple cylinders[14] as well.

S.d'Ambrosio, A. Ferrari, L.Galleani [15] presented his work to compute in-cylinder pressure and the essential combustion features such as start and end of combustion and the angle of crack shaft corresponding to half burned mass. The pressure has been recorded using piezoelectric[15] transducer. For diesel combustion the time and frequency analysis are also been applied. Using different values of crankshaft angular velocities and different types of engines running on gasoline, methane and other fuels are considered. For an internal combustion engine a design such as on-line estimator has purposed by Ibrahim Haskara and Larry Mianzo [16] which has the purpose of indicating torque and each cylinder pressure of combustion. FFT analyzer [16] is used and with accelerometer is used to measure the vibration which is important aspect of finding the valid reason for vibrations in the engine. The natural frequency and the characteristics of the motion have been mapped. Using Matlab and Simulink [17] established complete simulation model to study the complete features of motion. And the results thus obtained from free piston system shows that the system is forced vibrated with the effect of damping and stiffness. And the response in steady state with periodic excitation is converging thus representing the stability of the combustion in periodic manner. The simulation model includes thermodynamic, dynamic and thrust force models [17]. Md Wasim, Abhishek Bhandari [18] applied Taguchi method to determine what the best settings for a diesel engine piston are and the method is used in other fields of engineering as well. And it has done by the optimization of temperature and stresses using Minitab Software. And the piston shape, butanol fumigation, EGR[18], injection pressure, and injection timing affect the performance[18] The optimization strategy is effective in lowering operational costs and the streamlining of the processes used in design. For high temperature conditions Ansys 15 has been used for thermal analysis. The engine used is Light Duty DI diesel engine and the fuel used is diesel jatropha methyl ester (JME)[18] twenty percent of volume. A mathematical model including mixed lubrication for piston. The work done suggests how the friction, lubrication and piston motion is affected by the thermal distortion, elastic deformation, the profile[19] of the piston skirt, roughness and waviness of the surface of the cylinder bore and piston skirt.

And it is suggested by [20] that improvements are required in the structure of the test rig for accurate and cleaner measurements when the engine operates at higher speed such as above 800 rpm[20]. The experimentation token place for two types of piston configurations. And it is reviled from the results that the compression ring adds most of the fraction to the total friction all over the cycle of the piston. A subtraction method used suggests[20] using slightly oversized piston.

Xun Zhang, Hui Liu, Mahmoud Taha of Beijing institute of Technology [21] investigated the vibrations under the influence of active damping and these vibrations are produced by the engine ripple torque in power split hybrid electric vehicles (HEVs)[21]. Furthermore a simple model for dynamics of transmission is developed which simplifies the analysis of the transmission and makes the control of active vibration possible. And the active vibration controller which is basically works on the basis of filtered-x lead mean square (FxLMS) [21] type algorithm, this algorithm deals the vibration that is produced in the power split of hybrid electric vehicle. The results thus obtained for the steady-state conditions from the simulation and when the active damping algorithm is applied so the fluctuation of torque in the driveline shafts is decreased by 60 %[21] over various velocities and different types of modes of driving which shows the effectiveness of the demonstrated strategy.

The literature reviewed by [22] indicates that the piston skirt dynamics and also the Tribological performance which is basically the performance of the lubricant oil is supposed to be a complex engineering problem[22]. [23] used an algorithm NSGA 2[23] to optimize more than one objective function and in this case the objective function were for temperature for which the maximum value was required at piston top ring groove and second objective function was the equivalent maximum stress in upper part of the piston, for temperature and stress the best results are obtained when cooling gallery is close as possible to the top of the piston crown[23][24] carried out computational study on piston using Ansys and performed static and modal analysis of the piston of the engine. The engine specifications were WP10.290 Diesel engine, water cooled and four strikes engine additionally it is high pressure and electronically controlled common rail and supercharged intercooling, with 126 mm cylinder head, 213 W rated power, 2200 rpm rated speed[24], 600 rpm of idle speed and 1160 N-m maximum torque, maximum speed is 1200 rpm - 1600 rpm and this analysis involved only one piston and in static analysis has applied fixed constraints to model at inner wall of piston pin pedestal holes. And as far as loads are concerned instead of maximum burst pressure the average effective pressure has been used with value of 1.182 MPa. Using Ansys first sixth natural frequencies are found out and the mode shapes are shown in the figures[24].In 1950 IBM developed a programming language for scientific and engineering applications called Fortran the Fortran cones from formula testing system and ADAMS stands for Automated Dynamic Analysis of Mechanical systems is a simulation software and its solvers works on Fortran as well as C++. The general purpose of ADAMS is solving complex Multibody dynamics and the research carried out by 1 consist of establishment of dynamic model of piston cylinder system of an internal combustion engine (ICE) and their model has two parts, first is hydrodynamic lubrication and second part is the dynamics of Multibody system for that of piston cylinder system both parts are coupled together by ADAMS [25] and FORTAN subroutine. The deliverable of this work is that how the length of clearance between pistons sleeve and piston skirt dictates the lubricating characteristics of piston geometric parameters. Basically this is solving tow equations such as system dynamic equation and secondly average Reynolds equation and they are solved simultaneously with the help of ADAMS solver and FORTRAN subroutine[25]. [26] with critical loads of 300 N, 400 N and 500 N the change in friction coefficient with varying loads have been find out then spectrometric analysis has been carried out, for each case of critical loading and it has been observed that the amount of ferrum has increased dramatically above the critical load. The spectrometry results also contain the small particles (Ds) and large particles (D_l) in the lubricating oil and a trend has been shown how these two types of particles varies such as increasing critical load increases Ds relative to Dl[26] Then Friction between surfaces and quality of lubrication oil have been studied such that three groups with same surface roughness like 0.6 um -6.2 um have chosen for group 1 CD40 lubrication has been use in second group CD40[26] lubrication oil with MoS2 have been used and finally special running-in lubricant has been chosen and each group have critical load of 300 N, 400 N and 500 N respectively. Hence it has been concluded for [26] that spectrometric analysis on lubricating oil can work effectively to monitor the friction process in diesel engine, further it has showed that the type of quality of lubrication oil matters while formation of film and affects the friction process, and the experiments performed in this work are by Pin Pan[26] experiment machine. [27] mapped analytical model to find out the minimum film thickness of lubricating oil between piston ring and cylinder liner. This model is based on partial elastohydrodynamic lubrication (PEHL) which accounts for surface roughness. According to author previously used different types of lubrication like Elaso-Hydro_dynamic lubrication (EHL) and mixed lubrication (ML) didn't account for surface roughness and oil pressure, when these two factors are considered such as in (PEHL) [27]the minimum film thickness at (dead top center) of the cylinder liner becomes smaller then that calculated for EHL and greater then ML. The work done by [28] concentrates over the creation of tool for the study of the hydrodynamic lubrication for the rings and an experimental portion of this work using sensors for the measurement of the thickness of the lubrication film has also presented[28,29].

3. METHODOLOGY

As the reciprocating of the piston involves rotation of crank shaft and the reciprocation and rotation of connecting rod and reciprocation of the piston itself. In this vibrational analysis of piston two degree of freedom have been considered such as motion of the piston in x axis which is also called lateral motion of the piston and the motion along the y axis as shown in Fig. 10.

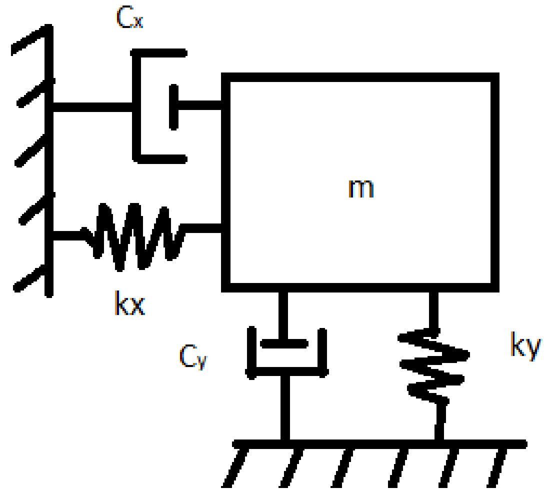


Fig. 10. Spring mass damper system

4. STIFFNESS:

To find the restoring force which causes the piston to restore to its mean position has been find out from the rotational energy of the crank shaft. In this approach during the expansion of gases in the cylinder work is done both on piston and crank shaft, the crank shaft stores some of the work in form of rotational energy which then helps the piston to restore to its mean position. And two components have been find out such as one for reciprocating and other for lateral motion of the piston.

$$\text{work done } = \text{RF}\theta \tag{1}$$

$$\text{change in rotational energy} = \frac{1}{2}I_c\omega^2 - \frac{1}{2}I_c{\omega_o}^2 \tag{2}$$

$$equevlent\ stored\ energy\ for\ restoring\ of\ piston\ \ E = \frac{1}{2}kx^2 \tag{3}$$

$$\text{k} = \frac{RF}{y^2} \tag{4}$$

$$\text{k}_y = kcos\alpha \tag{5}$$

$$k_x = ksin\alpha \tag{6}$$

5. DAMPING COEFFICENT

In this analysis only the viscous damping due to engine oil have been taken such as for x direction and y direction of the piston in the sleeve. The lubrication is hydrodynamic and Newtonian lubricant fluid has been used. The damping coefficient have find out using law of continuity and flow rate. As the piston during lateral motion displace the lubricant tangential to the piston dimeter and the shear force between the piston and cylinder is given by

$$Shear\ force = F = \pi rl\ d\tau \tag{7}$$

Where πrl half the area of the piston skirt, and this area is experiences the shear force due to the viscosity of the lubricant.

$$\tau = -\mu\frac{dv}{dy} \tag{8}$$

$$F = -\mu\pi rl\ \frac{d^2v}{dy^2}dy \tag{9}$$

$$pressure\ force\ on\ end\ elements = 2pl\ dy \tag{10}$$

$$where\ p = \frac{P}{\pi rl} \tag{11}$$

$$F = \frac{2P}{\pi r}dy \tag{12}$$

$$\frac{2P}{\pi r}dy = -\mu\pi rl\ \frac{d^2v}{dy^2}dy \tag{13}$$

$$\frac{d^2v}{dy^2} = -\frac{2P}{\pi^2r^2l\mu} \tag{14}$$

$$v = -\frac{2P}{\pi^2r^2l\mu}y^2 + y\left(\frac{Pd}{\pi^2r^2l\mu} - \frac{v_o}{d}\right) + v_o \tag{15}$$

Where d is the clearance and v_o is letral velocity of the piston and also

$$Q = \int_0^d 2lv\ dy \tag{16}$$

$$Q = \frac{2Pd^3}{3\pi^2r^2\mu} + v_o dl \tag{17}$$

$$also\ Q = Av_o = \pi rlv_o \tag{18}$$

$$P = \frac{v_o(\pi rl - dl)3\pi^2r^2\mu}{d^3} \tag{19}$$

$$also\ P = Cv_o \tag{20}$$

$$C_x = \frac{3(\pi r - d)\pi^2r^2l\mu}{d^3} \tag{21}$$

Using the same principles the translatory damping coefficient have been find out

$$C_y = \frac{A\mu}{d} \tag{22}$$

6. FORCES

Step force have been taken to find the two motions of the piston. This step force is a piece wise function of time which is zero for half rotation of the crank shaft and have a step value for the other half rotation of the crank shaft. This force has also two components for the two type of motions. The friction forces and inertia of the crank shaft have been ignored only the masses of connecting rod and the piston have been considered.

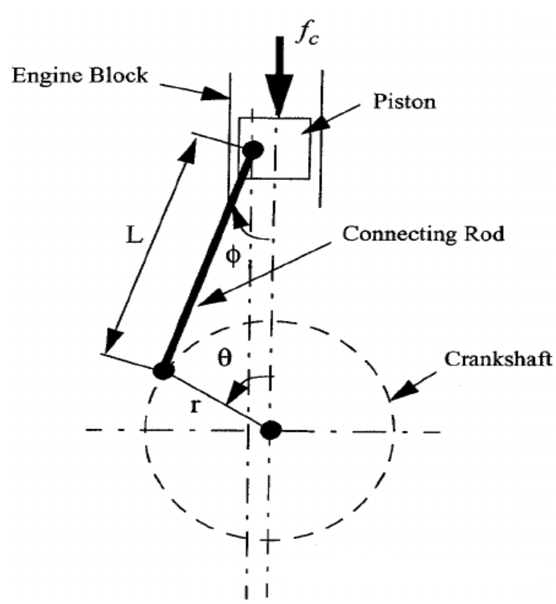


Fig. 11. Different Parameters of Piston Dynamics

$$Lsin\phi = r_c sin\theta \tag{23}$$

$$\phi = sin^{-1}\left(\frac{L}{r_c}\ sin\theta\right) \tag{24}$$

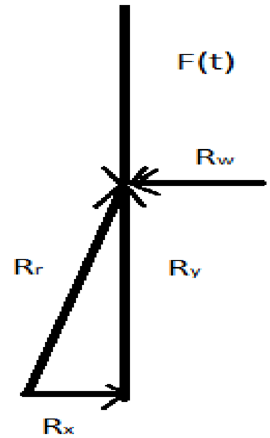


Fig. 12. Combustion forces and reaction from connecting rod and the cylinder wall.

$$R = \frac{T}{r} sin\theta \tag{25}$$

$$Rx = Rsin\ \phi \tag{26}$$

$$Ry = Rcos\phi \tag{27}$$

$$Rx = \frac{T}{r} sin\theta \sin\phi \tag{28}$$

$$Ry = \frac{T}{r} sin\theta \cos\phi \tag{29}$$

$$Fx(t) = Rx \tag{30}$$

$$Fy(t) = -Fy + Ry \tag{31}$$

7. EQUATION OF MOTION:

$$\begin{bmatrix} \mathrm{m} & 0 \\ 0 & \mathrm{mp} \end{bmatrix} \begin{bmatrix} \ddot{\mathrm{x}} \\ \ddot{\mathrm{y}} \end{bmatrix} + \begin{bmatrix} \mathrm{Cx} & 0 \\ 0 & \mathrm{Cy} \end{bmatrix} \begin{bmatrix} \dot{\mathrm{x}} \\ \dot{\mathrm{y}} \end{bmatrix} + \begin{bmatrix} \mathrm{kx} & 0 \\ 0 & \mathrm{ky} \end{bmatrix} \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \end{bmatrix} = \begin{bmatrix} \mathrm{Rx} \\ \mathrm{Ry} \end{bmatrix} \tag{32}$$

These are uncoupled equations and can be solved independently using Fourier Transform.

8. INPUT PARAMETERS

Table 3, List of Input parameters

Parameter	Symbol	Value
Crank Shaft inertia	I_c	1.3e-3 kg m^2
Ratio of connecting rod length	j	0.4
Connecting rod length	L	0.15m
Piston and piston pin mass	m_p	0.2kg
Connecting rod mass	m_r	0.35kg
Total mass of piston, piston pin and piston rings	M	0.33kg
Crank shaft radius	r_c	25mm
Piston skirt and cylinder liner clearance	d	0.5mm

9. IMPLIMENTATION

The response of each degree of freedom have been find out using Fourier transform because this problem include forcing function which is piece wise defined and Matlab have been used to code program for solutions. First of all the 127 cc motorcycle piston geometrical dimensions have been selected for the analysis and the values are fed into the stiffness mass and Damping coefficient equations find earlier and have been incorporated into the equation of motions and solved with Matlab.

$$a_j = \frac{2}{\tau} \int_0^{\tau} F(t) cosj\omega t dt, \;\; j = 0,1,2 \ldots. \tag{33}$$

$$b_j = \frac{2}{\tau} \int_0^{\tau} F(t) sinj\omega t dt, \;\; j = 1,2 \ldots. \tag{34}$$

$$x_p(t) = \frac{a_o}{2k} + \sum_{j=1}^{\infty} \frac{(\frac{a_j}{k})}{\sqrt{(1-(jr)^2)^2 + (2\xi jr)^2}} \cos(j\omega t - \phi_j) + \sum_{j=1}^{\infty} \frac{(\frac{a_j}{k})}{\sqrt{(1-(jr)^2)^2 + (2\xi jr)^2}} \sin(j\omega t - \phi_j) \tag{35}$$

These equations are used to find two solutions for each type of motion

10. RESULTS

For the step force of 5 N and time period of 0.1 sec there is major deflection of piston in lateral direction at 0.05 sec of time period and when force is zero at time period of 0.1 the piston regain its mean position.

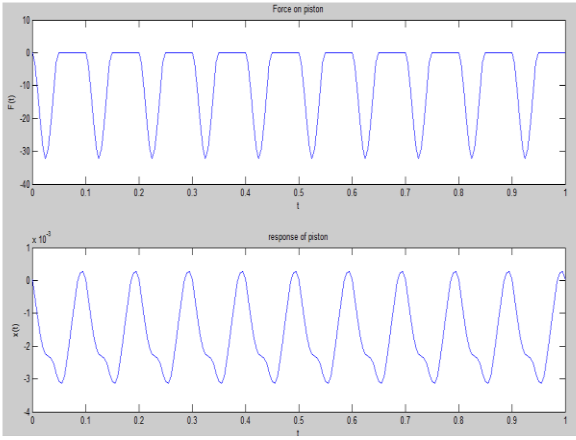


Fig. 13. Force on the Piston in x direction and its Piston Secondary/lateral response

The reciprocating response in y direction is shown in the following figure with the R_x component shown as F (t).

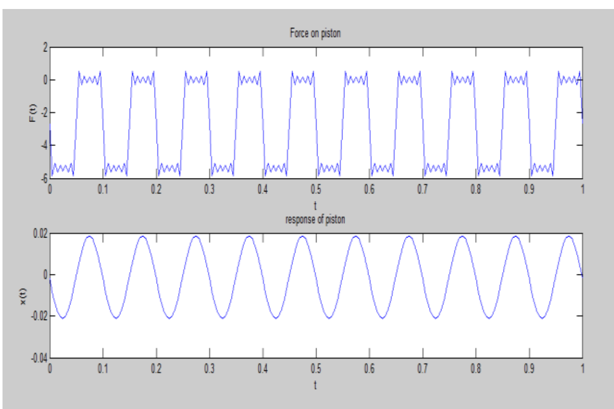


Fig. 14. Reciprocating force and piston response in y direction

11. CONCLUSION

This work shows that a secondary motion exists between tribological surfaces in which the surfaces move relative to each other and having hydrodynamic lubrication. As in piston cylinder arrangement, in which the reciprocatory motion is the primary motion along with this motion the lateral or secondary motion does exist as shown in Fig. 13.

The primary and secondary response of the piston have been obtained by modeling the piston as a system with two degrees of freedom and for both types the response have obtained independently using Fourier transform method.

The stiffness and damping due to the lubrication have been modeled and incorporated into the furrier equations and the response have been generated using Matlab with n = 10, T = 0.1.

The reciprocating or primary response is harmonic as in Fig. 14 while lateral or secondary response is different as in Fig. 13.

When two surfaces have clearance, having hydrodynamic lubrication and moving relative to each other the lateral response can be find out using the same model by changing the parameters governing the areas.

12. RECOMMENDATIONS:

A more realistic approach could be used to model the mechanism by including the inertia of crank shaft and there will be more accuracy in damping coefficient if we include the real piston skirt area and piston rings in modeling the lubrications.

NOMENCLATURE

Capital

P_{comp}	pressure during compression
P_{exp}	pressure during expansion
I_c	rotational inertia of the crank shaft
F	shear force
P	force causing hydrodynamic pressure
Q	rate of flow through the clearance space
C	damping coefficient
L	length of the connecting rod
R	reaction force
T	torque
M	total mass

Lowercase letters

k	stiffness of the system
r_p	radius of the piston
l	length of the piston
p	hydrodynamic pressure in the thin oil film
d	clearance between piston and cylinder
r_c	radius of the crank shaft
m_p	mass of the piston
m_r	mass of connecting rod
j	ratio of L and r_c
$\ddot{x}$	acceleration in x direction

$\ddot{y}$ acceleration in y direction

Greek capital symbols

ϕ angle of connecting rod with y axis
θ crank shaft angle

Greek lower case

μ oil viscosity
τ shear stress
ω angular velocity of crank shaft

Abbreviations

HL hydrodynamic lubrication
EHL elasto-hydrodynamic lubrication
PEHL partial elasto-hydrodynamic lubrication
ML mixed lubrication
VCR variable compression ratio
FLARE friction and lubrication analysis of reciprocating engines
NR newton-raphson
CAD computer aided design
MBF mass fraction burnt
JME jatropha methyle ester
HEVs hybrid electric vehicles
FxLMS filtered-x lead mean square
ADAMS automated dynamic analysis of mechanical systems
ICE internal combustion engine

ACKNOWLEDGEMENTS

We are extremely thankful to the Mechanical and Aerospace Engineering department of Air University Islamabad for the opportunity, resources and guidance to conduct this work.

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