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Two Flaws of the Net Present Value Criterion

Muhammad Mazhar Iqbal*

Abstract

For project evaluation, the net present value (NPV) criterion is the most preferred one. It attaches a pre-fixed opportunity cost to initial investment. Therefore, it ranks projects by the amount of profit. It favors bigger size projects. If supply of capital for a country is limited, then individual firms' project selection by the NPV criterion may lead to less than potential level of output, a flaw of this criterion. The other flaw is that its formula does not account for the opportunity cost of initial investment if a project is to be financed by owners' capital. Consequently it overestimates NPV of such projects.

Keywords: Net present value, internal rate of return, project evaluation, productive efficiency.

1. Introduction

A firm can materialize only a few out of many competing projects because some of them could be mutually exclusive technically while others could be mutually exclusive financially. Two projects are said to be technically mutually exclusive if execution of one project precludes execution of the other. For example, construction of a shopping plaza on a given plot precludes construction of residential apartments on the same plot. Two projects are said to be financially or economically mutually exclusive if execution of one project precludes execution of the other at the given time due to financial or other economic constraints. For example, construction of a shopping plaza on one plot may preclude construction of residential

*Muhammad Mazhar Iqbal, Associate Professor, School of Economics, Quaid-i-Azam University Islamabad Pakistan. Email at: mmiqbal@qau.edu.pk

apartments on another plot at the same time due to shortage of funds (Ng and Beruvides, 2015). In this paper, however, the word 'competing' is being used for all mutually exclusive projects whether technically or financially.

Therefore, to select one from two competing projects, firm managers need a sound criterion to justify their choice. For this purpose, there are many criteria such as the payback period (PBP), the accounting rate of return (ARR), the internal rate of return (IRR) and the NPV to choose from (Bearley, et al., 2010; Copeland, & Weston, 1988; Levy, & Sarnat, 1990). However, the IRR and NPV methods generically known as discounted cashflow (DCF) methods have dominated the others and are therefore widely discussed (Kierulff, 2012; Karathanassis, 2004; Meyer, 1979; Brigham, 1975; Dudley, 1972; Gould, 1972; Wright, 1962; Roberts, 1957; Renshaw, 1957; Solomon, 1956; Alchian, 1955; and Lorie, & Savage, 1955). Out of these two methods, the NPV criterion has been more popular (Bearley, et al., 2010; Levy, & Sarnat, 1990; Roberts, 1957; Renshaw, 1957; Alchian, 1955 and Lorie, & Savage, 1955). The reason is that it takes into account all cash flows of every competing project whereas the PBP criterion considers only earlier cash flows; it attaches a positive time value to money whereas the PBP and ARR methods do not; it is useful to evaluate both conventional and non-conventional projects whereas the IRR criterion is good to evaluate mainly conventional projects; and it aims at maximization of shareholders' value whereas the IRR method aims at maximization of the rate of return on total invested funds whether raised by equity or by debt financing.

However, the IRR criteria performs better if a firm has access to limited funds or it faces 'hard' capital rationing to finance its projects. In such a situation, the overall NPV of a firm from all of its projects comes out greater if each of its projects is selected by the IRR criterion rather than by the NPV criterion (Bearley, et al. 2010; Matson, 1999; Copeland, &Weston, 1988; Levy, & Sarnat, 1990). This weakness of the NPV criterion is though illustrated in some textbooks, yet it is not discussed much with the proviso that in the presence of a developed financial system, almost all firms can raise as many funds externally as they want. For example, Matson (1999) states, "For some years, the typical research focus has been on the treatment of risk, assuming perfect capital markets in which there is no room for credit rationing." In the same vein, a limited supply of monetary capital for a country, which seems quite plausible, is also not paid due attention in theoretical discussions. This trend, in fact, goes against the basic assumption of economics that resources are scarce and wants are unlimited. The IRR criterion is also preferable over the NPV criterion if the objective of a firm is to maximize its rate of growth rather than shareholders' value in a given time period (Dorfman, 1981). Many authors have tried to reconcile the results of IRR and NPV criteria (Johnstone, 2010; Osborne, 2010; Liu, & Wu, 1990)

The first objective of this research is to investigate the aftermaths of limited supply of capital for a country for its productive efficiency assuming that all firms in such a country use only the NPV criterion for project evaluation. In this regard, it is assumed as in textbooks of corporate finance that individual firms can raise as many funds externally either by issuing equity or by borrowing as they want. However, a condition is imposed that it is possible if the total demand for funds of all firms in the country remains equal to or less than the limited supply of capital for the country; otherwise a firm which applies for external funds before another one is able to get its required funds until the supply of funds lasts. Therefore, on the reasoning of analogy of a firm facing 'hard' rationing of funds, this research hypothesizes that a country with a limited supply of funds may end up with a smaller than potential level of NPV.

The other flaw of the NPV criterion is that its formula does not inculcate the opportunity cost of initial investment of a project if it is to be financed by owners' equity. Since a firm is not allowed to go public without raising a specified amount of owners' capital and after its incorporation and similarly a non-incorporated firm usually does not finance all or most of its investment projects by borrowed money in any market economy, therefore it can be argued that this lapse of NPV formula may result in selecting a number of low yielding projects. Therefore, the other objective of this research is to illustrate this flaw of the NPV criterion mathematically as well as numerically.

The scheme of this paper is that the next section describes the NPV criterion; its formula and its main features. Section three illustrates numerically that by using the NPV criterion, firm managers tend to select relatively bigger size projects even if they are less efficient. This tendency of managers of individual firms may lead to overall low NPV on investment of the limited supply of monetary capital for a country. Section four describes that by not incorporating opportunity cost of equity capital in the familiar NPV criterion; corporate managers select less efficient and, in some cases, even value-reducing projects. The last section is reserved for conclusions.

2. Description of the NPV Criterion

To understand the basics of NPV criterion, only conventional projects are considered in this study and it is assumed that cash flows of all competing projects are known with certainty though the results should not change much if nonconventional projects are considered as well and the assumption of certainty is relaxed. A conventional project requires outlays in earlier periods and generates cash flows in later periods and a nonconventional project may take any form; for example it may give return in earlier periods and require cost in later periods or the sequence of its cost and return periods is switched more than once. Many authors have focused only on nonconventional projects (Kulakov, & Kastro, 2015; Kulakov, & Kulakova, 2013; Hartman, & Schafrick, 2004).

The formula to calculate npv of a conventional project is:-

$$npv = -I_0 + \sum_{t=1}^{n} (CF_t)/(1 + i)^t \qquad (1)$$

where I_0 shows the cost of project or initial investment that is to be incurred in the zero period, CF_t shows a net cash flow that the underlying project generates at time t that varies from 1 to n, i denotes the risk-free interest rate. The interest rate on short term government securities is generally taken as the risk-free interest rate and it represents the time value of money (Bearley, et al., 2010; Fabozzi, et al., 2003). Since different interest rates have a tendency to move in unison, therefore economists frequently lump together and refer to 'the' market interest rate (Mishkin, & Eakins, 2011). Practically i is used to calculate, on one hand, the opportunity cost of initial investment and, on the other hand, the reinvestment income for cash flows to be generated before the final one. It means that in addition to explicitly given initial cost and future cash flows of a project, the NPV criterion takes into account the imputed opportunity cost of the initial investment and the imputed reinvestment income for all its future cash flows before the final one. The imputed opportunity cost of initial investment is calculated as:-

$$\text{Imputed opportunity cost} = I_0(1 + i)^n - I_0) \tag{2}$$

And the reinvestment income of all future cash flows before the final one is imputed as:-

$$\text{Imputed reinvestment income} = \sum_{t=1}^{n} ((CF_t)(1 + i)^{n-t} - CF_t) \tag{3}$$

Hence, the sum of initial investment and its imputed opportunity cost comes out:-

$$\text{Total cost up to period } n = I_0(1 + i)^n \tag{4}$$

and the sum of explicitly given cash flows and their imputed reinvestment income comes out:-

$$\text{Total revenue up to period } n = \sum_{t=1}^{n} (CF_t)(1 + i)^{n-t} \tag{4}$$

Hence the net future value (NFV) of the project in period n comes out:-

$$nfv = - I_0(1 + i)^n + \sum_{t=1}^{n} (CF_t)(1 + i)^{n-t} \tag{5}$$

Then discounting this NFV by (1+ i)n, the familiar NPV formula is obtained as given in equation (1) above.

In this formula, the calculated NPV of any project is a negative function of the market interest rate. It means that before calculation of NPV, a numerical value for i must be assumed. The functional dependence of NPV on i is illustrated numerically assuming a hypothetical project that has I_0 = \$100, CF1 = \$10 and CF2 = \$110 in Table 1.

Table 1
Relationship between NPV and Interest Rate

Interest Rate (in %)	0	2	4	6	8	10	12	14
NPV (in \$)	20	15.53	11.32	7.33	3.57	0	-3.38	-6.58

As can be seen from the second row of the Table, NPV of this project is \$20 when interest rate is zero, it remains positive at any interest rate less than 10 percent, it is zero when interest rate is 10 percent and it becomes negative at any interest rate greater than 10 percent. The relationship between the calculated NPV and i can also be illustrated graphically as in Figure 1.

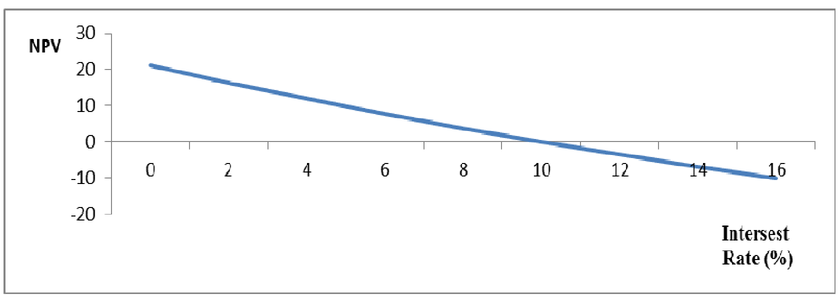

Fig. 1 Illustration of Functional Dependence of NPV on Interest Rate

That particular i discounting by which the present value of future net cash flow becomes exactly equal to the initial investment is called the IRR of the project. The value of IRR does not depend on exogenously given interest rate but only on the initial investment and future cash flows. That is why it is called the 'internal' rate of return and is often viewed as a measure of productive efficiency of a project (Hartman, & Schafrick, 2004).

A single project which has a positive NPV at the given interest rate is worth undertaking and out of many competing projects, the one which has the highest NPV is ranked at the top and the one which has the lowest but positive NPV is ranked at the bottom. However, ranking of competing projects does not remain the same at different interest rates; rather it changes. If project ranking by the IRR criterion is taken as the reference point, then project ranking by the NPV criterion undergoes a substantial change at lower interest rates particularly if initial costs of competing projects are significantly different as illustrated in the next section.

3. Selection of Less Efficient Projects as the NPV Criterion Maximizes the Amount not the Rate of Profit

In the NPV criterion, a pre-fixed opportunity cost is attached to the initial investment of every project irrespective of its size. If future cash flows to be generated by the project are known with certainty as assumed in this research, then the interest rate as such is used to estimate the opportunity cost, otherwise a risk premium that can be found by different methods is added to the market interest rate (Espinoza, 2014; Bearley, et al., 2010; Fabozzi, et al., 2003; Levy, & Sarnat, 1990). Indirectly it means that every project can be financed by borrowed money or a firm can borrow as many funds at the market interest rate as it wants. Therefore, the objective of corporate managers becomes maximization of the amount of profit rather than maximization of the rate of profit on total invested funds. Since the amount of profit is a direct function of the size of a project, therefore, taking IRR of a project as a gauge of its productive efficiency, it can be proven easily that the NPV criterion leaves out many efficient projects which are smaller in size but are equally feasible. It usually happens when the market interest rate is relatively low. For illustration, two mutually exclusive projects are considered which require different initial costs as given in Table 2 below.

Project A is smaller in size but is more efficient than project B as IRR of the former is 15 percent and that of the latter is 10 percent as shown in column 4. However, column 5 shows that by the NPV criterion, at i = 5 percent both projects are equally good and columns six shows that at i = 3 percent, their ranking is just the opposite of that by the IRR criterion. The NPV of project B is $13.59 and that of project A is $11.65.

Table 2
Contradictory Ranking by the IRR and NPV Criteria at Low Interest Rates

Project\Period	0	1	IRR	NPV i=5%	NPV i=3%
A	-$100	$115	15%	$9.52	$11.65
B	-$200	$220	10%	$9.52	$13.59

Having proved that by using the NPV criterion in a regime of low interest rates, firm managers tend to select relatively bigger size projects even if they are productively less efficient, the next task is to investigate the consequences of firm-level choice of relatively bigger-size projects for overall productive efficiency of an economy that has a limited supply of capital. In textbooks of microeconomics, productive efficiency of a hypothetical economy is illustrated through production possibility frontier (PPF) assuming that only two goods X and Y are produced by given endowment of two inputs capital and labor. All points on the PPF show maximal utilization of capital and full employment of labor and all points inside the PPF show either under-utilization of capital or unemployment of labor or both and thus represent a case of productive inefficiency (Varian 2009; Pindyck and Rubinfeld 2009).

To show the impact of a firm's choice of projects upon productive efficiency of a country, let us assume that either the state has restricted capital mobility across the borders of this country or foreign direct investment in the country is a tiny fraction of its total investment and demand of financial instruments denominated in local currency by foreigners is also negligible; its supply of capital in monetary terms is limited to $200 that can be either loaned out or provided on equity basis or both to domestic firms through efficient capital markets on first come first served basis; it has only 2 identical incorporated firms which acquire all of their required funds externally; and each of these firms come across only 2 competing projects as described in Table 2 above, out of which it has to choose only one project. With these simplifying assumptions, first the choice of an individual firm is looked at by assuming that it selects its project by the NPV criterion at i = 3 percent and then analyze consequences of the firm-level choice for productive efficiency of the country. By the NPV criterion, each firm selects project B as shown in Table 2 above. Hence, the total amount required by both firms to undertake their respective project B will be $400 that exceeds the given supply of $200 for the economy. As a result, only one firm will be able to raise its required funds on first come first served basis. Consequently the other firm being late to apply for external financing will be deprived of external funds totally. Hence the total NPV of the country will be $13.59. On the other hand, if each firm had used the IRR criterion, then each firm would have selected project A. The funds required to materialize project A are $100. Hence, the total required amount by both firms would have been $200 that is exactly equal to the given supply for the economy. As a result, each firm would have been able to raise its required funds on first come first served basis. Hence the total NPV of the country would have been $23.30 that is almost twice as much as the one obtained under the assumption that each firm selects its project by the NPV criterion.

This numerical exercise proves the hypothesis of this research that by the NPV criterion, firm managers are biased toward bigger size projects at lower interest rates even if these projects are less efficient. It means that the NPV criterion leads to inefficient utilization of the limited supply of monetary capital for a country that is tantamount to productive inefficiency in the country. That is, the country does not produce the highest possible level of output of various goods and services because its limited capital is not used to finance smaller size but more efficient projects which are equally feasible.

4. Selection of Less Efficient Projects as the NPV Criterion Leaves Out Opportunity Cost of Equity Capital

If a firm plans to finance a given project by borrowed money, then its initial investment (I_0) is virtually zero because the firm does not incur its owners' funds. Consequently there is no need to impute the opportunity cost of initial investment. Rather the actual interest payment in each period ($i\times I_0$) is treated as its opportunity cost. Moreover, assuming that the firm has to pay back its debt in lump sum after the life of project that is the nth period, the NPV of this project can be worked out as: -

$$npv = \sum_{t=1}^{n} (CF_t)/(1+i)^t - \sum_{t=1}^{n} (i\times I_0)/(1+i)^t - (I_0)/(1+i)^n \qquad (6)$$

Summing up the geometric series that emerges from the second term:-

$$\sum_{t=1}^{n} (i\times I_0)/(1+i)^t = I_0 - (I_0)/(1+i)^n \qquad (7)$$

Hence, simplification of equation (6) using equation (7) results in equation (1).

It proves that the commonly used NPV formula as given in equation (1) is meant to evaluate actually those projects which are to be financed by debt. It means that if a firm plans to finance its project by owners' capital, either by retained earnings or by issuing new shares or by both, then accounting for the opportunity cost of initial investment its NPV should be calculated as:-

$$npv = -I_0 + \sum_{t=1}^{n} (CF_t)/(1+i)^t - \sum_{t=1}^{n} (i\times I_0)/(1+i)^t \qquad (8)$$

It must be less than that calculated by equation (1) by the amount of imputed opportunity cost of initial investment that is the last term in equation (8).

For numerical illustration, the same project is considered as described in Table 1 is to be financed by owners' equity. Then its NPV comes out in negative (-3.67). The difference in two NPVs comes out 11.00 that is the present value of imputed opportunity cost of initial investment for the 2-year life of this project. It follows that if initial investment of a project is to be financed by a mix of debt and equity, say α percent by owners' equity and $(1 - \alpha)$percent by debt, then the NPV formula should be amended as:-

$$npv = -I_0 + \sum_{t=1}^{n} (CF_t)/(1 + i)^t - \alpha \sum_{t=1}^{n} (i \times I_0)/(1 + i)^t \qquad (9)$$

This illustration clarifies that using the familiar NPV formula, incorporated firms routinely overestimate NPVs of selected projects because most of them are, in fact, financed by mix of debt and equity.

5. Conclusion

Managers of incorporated firms routinely come across many competing projects; out of which they have to choose a few. Therefore, they need a sound criterion to evaluate competing projects on the table. There are many criteria for this purpose but the NPV criterion is generally preferred over all others. The reason is that it assumes a positive time value for money that is the market interest rate. Practically it means that in addition to explicitly given initial cost and future cash flows of a project, corporate managers need to consider the imputed opportunity cost of the initial investment and the imputed reinvestment income of all its future cash flows before the final one.

However, by determining the opportunity cost of initial investment by an exogenously given interest rate before the calculation of NPV, it is indirectly claimed that the underlying project irrespective of its size can be financed by borrowed money at the given market interest rate. Therefore, the objective of firm managers becomes maximization of the amount of NPV that is a positive function of the size of a project. That is why the NPV criterion is by default, biased toward bigger size projects even if they are less efficient according to the IRR criterion. This bias of the NPV criterion may not be worrisome if supply of funds in a country is unlimited. That is, all firms having projects with positive NPV are able to finance them. In reality, however this is not the case. The supply of funds in any country has not been so abundant that all firms can finance their all profitable projects by borrowing money at the market interest rate.

Therefore, considering a hypothetical economy having a limited supply of funds and having only two incorporated firms which come across two similar competing projects of different sizes, it is been shown that each of them selects the bigger size project. As a result, the total demand for funds exceeds the limited supply of funds for the country. Then on the principle of first come first served, only one firm is able to raise required funds even though capital markets are assumed to be functioning efficiently in the country. As a result, the limited supply of capital for the country is utilized to finance the bigger size but less efficient project which materializes a lower than the potential level of output for the country.

The other flaw is that the NPV formula is basically meant to evaluate those projects which are to be financed totally by borrowed money. The initial investment of such a project is, in fact, zero for the firm. Therefore, it is no more needed to impute the opportunity cost of borrowed money; rather the actual interest paid periodically for the borrowed funds serves as its opportunity cost. However, the discounted value of periodical interest payments and discounted value of the payment of principal in the nth period is mathematically equal to the initial investment or the borrowed amount. That is why the initial investment is usually shown as if paid by the firm from its retained earnings, but periodical interest payments and payment of the borrowed money in the nth period are not deducted from future cash flows.

It means that if a project is to be financed totally or partially by owners' equity, then its opportunity cost that is the sum of forgone periodical interest earnings until the life of project have to be taken into account. Consequently, the formula to calculate NPV has to be amended as suggested above in equations (8) and (9). As a result, the value of NPV comes out significantly less than that calculated by the standard formula as given in equation (1). It means that the use of formula given in equation (1) generally overestimates the true NPV of projects to be financed totally or partially by owners' equity.

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