

The Kinked Demand Model and the Stability of Cooperation *

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Abstract

This paper revisits a particular behaviour for firms competing in imperfect competitive markets, underlying the well-known model of kinked demand curve. We show that under some symmetry and regularity conditions, this asymmetric behaviour of firms sustains monopoly pricing, and possesses therefore some "rationality" interpretation. We also show that such behaviour can be generalized and interpreted as a norm that sustains efficient outcomes in a more general class of symmetric games.

Keywords: *Kinked Demand, Symmetric Games, Social Norms of Behavior.*

1. Introduction

This paper focuses on the postulated behavior of firms competing in imperfect competitive markets, first theorized in the late 30's by a number of well-known economists, Robinson (1933) and Sweezy (1939), and best known as the "kinked demand model". This basically predicts an asymmetric behavior of firms in response to a price change, each expecting its rivals to be more reactive in matching its price cuts than its price increases. This prediction has been empirically tested by Hall and Hitch (1939) and later by

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Bhaskar *et al.* (1991), extensively criticized as not grounded in rational behavior by Stigler (1947), Domberger (1979), Reid (1981) and more recently analyzed in a dynamic settings by Marschak and Selten (1978), Bhaskar (1988), Anderson (1984), Maskin and Tirole (1988), among the others.

In this paper we add to this debate by showing that this behavioral rule possesses strong stability properties and, therefore, may sustain firms' collusion. In particular, in a symmetric and monotone market, we prove that, if every firm adopts and expects a simple kinked-demand norm of behaviour (KD), the symmetric collusive outcome (i.e. monopoly pricing) constitutes an equilibrium. We show that this result is rather robust and can be extended to all n -person symmetric strategic form games: a KD norm of behaviour always makes the symmetric efficient strategy profile (the one maximizing the sum of all players' utility) stable. Moreover, we show that under some additional standard assumptions on players' payoff functions, a slightly stronger norm of behaviour (implicitly implying a norm of reciprocity) makes the efficient outcome the only stable outcome of the game. Finally, it can be shown that KD norm of behaviour reflects a rational (maximizing) behaviour whenever players' actions are strategic complements in the sense of Bulow *et al.* (1982).

The paper is organized as follows: Section 2 sketches the paper idea in a classical two-firm kinked demand model. Section 3 introduces a more general game-theoretic setting. Section 4 provides the conclusion.

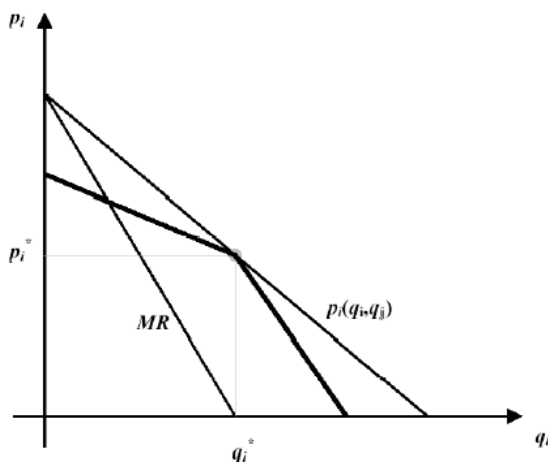
2. The Kinked Demand Model

The original idea of the kinked demand model (Robinson 1936, Sweezy, 1939) is based on the assumption that firms competing in a common market would react to changes in rivals' prices in an asymmetric manner. Specifically, when a firm increases its price it expects the other firms to raise their price comparatively less (under-reaction); when a firm lowers its price, conversely, it expects the others to further reduce their price (over-reaction).

This expected behavior generates a perceived demand with a "kink" at the original price levels (see Figure 1).¹

The main insight of this note can be illustrated by means of a simple case of two firms competing in prices in a common imperfectly competitive market with differentiated goods. Suppose prices are set at collusive levels (p_1^*, p_2^*) , i.e., in order to maximize the sum of firms' profits.

Figure 1. Kinked Demand Model



The kinked demand model assumes that the following behaviour, here expressed as a reaction function $k_j(p_i)$ for every $i = 1, 2, j \neq 1$, would prevail in case of deviation from collusive pricing:

$$\text{if } p_i' > p_i^*, \text{ then } k_j(p_i') \leq p_i' \text{ for every } p_i' \neq p_i^*$$

$$\text{if } p_i' < p_i^*, \text{ then } k_j(p_i') \leq p_i' \text{ for every } p_i' \neq p_i^*$$

Note that no presumption of best response (rationality) is assumed for $k_j(\cdot)$. The main point of this paper is that if firms adopt and expect the above behaviour, then any deviation from the collusive pair of prices

¹ The two firms i and j in Figure 1 are assumed to set their quantities so to equate market marginal revenue (MR) to their marginal costs (MC), here assumed constant and equal to zero.

(p_1^*, p_2^*) is prevented, and collusion is a stable outcome. To see this, suppose one firm, say firm 1, decides to deviate from the pair of strategies (p_1^*, p_2^*) , to improve upon its profit, that is, $\pi_1(p_1', k_2(p_1')) > \pi_1(p_1^*, p_2^*)$, where every firm's profit can be intended in the usual way as revenue minus cost.² It is well known that under price competition the effect of an increase in rivals' prices yields a positive effect on every other firm's profit, i.e., $\partial \pi_i / \partial p_j \geq 0$ (positive spillovers). Thus, if

$$\pi_1(p_1', k_2(p_1')) > \pi_1(p_1^*, p_2^*),$$

it must be that

$$\pi_1(p_1', p_2') \geq \pi_1(p_1', k_2(p_1')) > \pi_1(p_1^*, p_2^*).$$

By symmetry, $\pi_1(p_1', p_2') = \pi_2(p_1', p_2')$, and then

$$\sum_{i=1}^2 \pi_i(p_1', p_2') > \sum_{i=1}^2 \pi_i(p_1^*, p_2^*)$$

contradicting the efficiency (for the firms) of the perfectly collusive outcome. The same result obviously holds when it is firm 2 to deviate. This implies that if all firms expect a kinked demand response from all other firms, that is, when a *kinked social norm* is the established norm of behavior for all firms, no profitable deviations are possible from the perfectly collusive outcome (monopoly pricing).

Interestingly, the result extends to the case in which the firms set quantities instead of prices. The kinked demand model now dictates the following (for every $i = 1, 2$, $j \neq i$ and every feasible quantity):

² The argument is general and does not depend on the specification of the firms' profit, representable as $\pi_1(p_1, p_2) = p_1 q_1(p_1, p_2) - c_1(q_1(p_1, p_2))$.

if $q'_i > q_i^*$, then $k_j(q'_i) \geq q'_i$ for every $q'_i \neq q_i^*$

if $q'_i < q_i^*$ then $k_j(q'_i) \geq q'_i$ for every $q'_i \neq q_i^*$

where q'_i indicates any feasible quantity different from q_i^* , and $k_j(q'_i)$ the quantity set in response by its rival. Again, it is well known that under quantity competition the effect of a rise in the competitor's quantity yields a negative effect on every firm's profit (negative spillovers), that is, $\partial \pi_i / \partial q_j \leq 0$, since it lowers the market price $p(q_1, q_2)$. Hence, if firm 1 profitably deviates from the pair of strategies (q_1^*, q_2^*) and $\pi_1(q'_1, k_2(q'_1)) > \pi_1(q_1^*, q_2^*)$, it follows that

$$\pi_1(q'_1, q'_2) \geq \pi_1(q'_1, k_2(q'_1)) > \pi_1(q_1^*, q_2^*)$$

Since, by symmetry, $\pi_1(q'_1, q'_2) = \pi_2(q'_1, q'_2)$,

$$\sum_{i=1}^2 \pi_i(q'_1, q'_2) > \sum_{i=1}^2 \pi_i(q_1^*, q_2^*)$$

which, again, contradicts the efficiency of the pair of strategies (q_1^*, q_2^*) .

In the next section we show that this result holds in the general class of symmetric strategic games.

3. A General Setting

The result sketched above does not rely on the specific structure of imperfect competition, but only on the asymmetry of the assumed reaction to changes in players strategies, and on some built-in symmetry. The aim of this section is therefore to give a precise statement of the result in a larger class of games that still preserve the needed symmetry and monotonicity.

In this class of games, players are endowed with the same strategy space and perceive symmetrically all strategy profiles of the game. Moreover, players' payoffs possess a monotonicity property with respect to their opponents' choices. Although very specific, this setting still covers many well known

economic applications (as Cournot and Bertrand oligopoly, public goods games and many others).

We denote a monotone symmetric-player game in strategic form as a triple $G = \langle N, (X_i, u_i)_{i \in N} \rangle$, in which $N = \{1, \dots, i, \dots, n\}$ is the finite set of players, X_i is player i 's strategy set and $u_i : X_1 \times \dots \times X_n \rightarrow \Re$ is player i 's payoff function. We assume that each strategy set is partially ordered by the binary relation \equiv . We assume the following:

P.1 (Symmetry) $X_i = X$ for each $i \in N$. Moreover, for every $i \in N$ and any arrangement of the strategy indexes,

$$u_i(x_i, x_{-i}) = u_1(x_1, x_2, \dots, x_n) = u_2(x_2, x_1, \dots, x_n) = \dots = u_n(x_n, x_2, \dots, x_1).$$

P.2 (Monotone Spillovers) For every $i, j \in N$ with $j \neq i$, and every $x_j^1 \equiv x_j \equiv x_j^2$ we have either "positive spillovers" (PS)

$$u_i(x_{-j}, x_j^1) \geq u_i(x_{-j}, x_j) \geq u_i(x_{-j}, x_j^2),$$

or "negative spillovers" (NS)

$$u_i(x_{-j}, x_j^1) \leq u_i(x_{-j}, x_j) \leq u_i(x_{-j}, x_j^2),$$

where $x_{-j} = (x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_n)$.

A strategy profile x is symmetric if it prescribes the same strategy to all players. A *Pareto Optimum* (**PO**) for G is a strategy profile x^o such that there exists no alternative profile which is preferred by all players and is strictly preferred by at least one player. A *Pareto Efficient* (**PE**) profile x^e is a **PO** profile that maximizes the sum of payoffs of all players in N .

Let us now introduce the notion of a generic *social norm of behavior*.³

Definition 1 (*Social norm of behavior*). We say that the social norm of behavior $\sigma : X \mapsto X^{n-1}$ is active in G if every player $i \in N$ deviating from a given profile of strategies $x \in X_N$ by means of the alternative strategy $x'_i \in X_i$ such that $x'_i \neq x_i$, expects the response $\sigma_{N \setminus i}(x'_i)$ from all players $j \in N \setminus i$.

Finally, let us introduce a general definition of stability of a strategy profile in our game G , under any arbitrary social norm of behavior.

Definition 2 A strategy profile $x \in X_N$ is stable under the social norm \square if there exists no $i \in N$ and $x'_i \in X_i$ such that

$$u_i(x'_i, \sigma_{N \setminus i}(x'_i)) > u_i(x) \quad \forall i \in N.$$

We are interested in the *Kinked Social Norm* (KSN) of behavior, defined as follows:

Definition 3 (*Kinked Social Norm*) A *Kinked Social Norm of behavior* (KSN) k is defined as follows: for each $i \in N$, and $x'_i \in X_i$

$$\mathbf{k}_{N \setminus i}(x'_i) = (\forall j \in N \setminus i, x_j \in X_j \mid x_j \neq x'_i)$$

under positive spillovers (PS) and

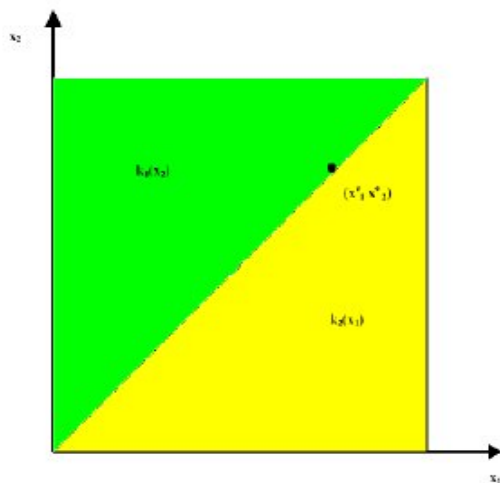
$$\mathbf{k}_{N \setminus i}(x'_i) = (\forall j \in N \setminus i, x_j \in X_j \mid x_j \equiv x'_i)$$

under negative spillovers (NS).

³The emergence of norms of behaviour can be viewed as arising from the evolution of shared expectations into prescriptions and then into norms of behaviour (see, for instance, Lewis 1969, Bicchieri, 1990 and Castelfranchi *et al.*, 2002). Once established within an organization, i.e.a firm, a set of norms ends up to represent its *corporate culture* (see, for instance, Brown (1995)).

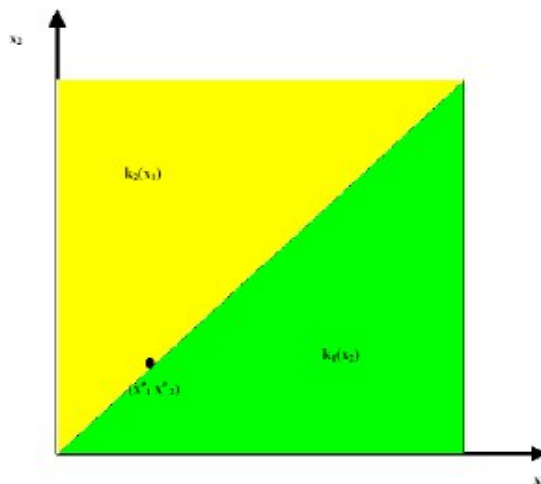
Note that, according to the definition above, the KSN imposes to all agents in $\mathcal{N} \setminus i$ to play a strategy lower (greater) or equal than the strategy played by the deviating player i under positive (negative) spillovers. Pictures 2 and 3 below represent graphically the KSN for a unidimensional closed and bounded strategy space $X_i, i = 1,2$, in the two-player case under either positive (figure 2) and negative spillovers (figure 3). In both pictures, the darker (brighter) area represents the KSN for player 1 (player 2) under either positive or negative spillovers.

Figure 2. KSN in the Two-Player Positive Spillover Case



In both pictures, the darker (brighter) area represents the KSN for player 1 (player 2) under either positive or negative spillovers.

Figure 3. KSN in the Two-Player Negative Spillover Case



The vector $x^e = (x_1^e, x_2^e)$ represents the symmetric PE strategy profiles in the two cases. Note that behind the KSN of behavior there is no presumption of rational behavior and players' reactions may easily not correspond to their best reply mappings (see below for a brief digression on this point).

We are now ready to present the main results of the paper.

Proposition 1: *Let conditions P1-P2 hold on G. Then, under the Kinked Social Norm of behavior(KSN), all symmetric Pareto efficient strategy profiles of G are stable.*

Proof: *See Appendix.*

Proposition 1 simply tells us that if the expected behavior of players in the event of a deviation from an efficient strategy profile is described by the kinked social norm, then every such efficient profile, if reached, is stable. In terms of imperfect competition, the expected kinked behavior of firms makes collusion a stable outcome.

The example below makes clear that stable inefficient (and asymmetric) outcomes cannot be ruled out without adding more structure to the above analysis.

Example 1: (2-player symmetric and positive spillovers game)

	<i>A</i>	<i>B</i>	<i>C</i>
<i>A</i>	4,4	2,3	1,2
<i>B</i>	3,2	2,2	1,2
<i>C</i>	2,1	2,1	1,1

In this game we assume that players' strategy can be ordered and, e.g., $A \succ B \succ C$, therefore the game respects both P.1 and P.2, with positive spillovers (PS). In this game, (A, A) , the PE strategy profile, is obviously stable under KSN. If, say player 1 deviates playing B , KSN implies $k_2(B) = \{B, C\}$ and player 1 ends up with a lower payoff than before, since $u_1(A, A) > u_1(B, B) > u_1(B, C)$. By symmetry, the same happens to player 2. However, asymmetric inefficient strategy profiles as (B, A) , (A, B) , (C, A) and (A, C) are also stable under KSN, given that $u_1(B, A) > u_1(A, B) > u_1(A, C)$ and $u_1(C, A) > u_1(B, B) > u_1(B, C)$ and the same for player 2.

To strengthen the result of proposition 1 and rule out inefficient stable outcomes, we add the following assumptions on the structure of G .

P3. Each player's strategy set is a compact and convex subset of the set of real numbers.

P4. Each player i 's payoff function $u_i(x)$ is continuous in x and strictly quasiconcave in x_i .

Under these additional conditions, Lemma 1 in the appendix shows that there is a unique Pareto Efficient strategy profile of G , and it is

symmetric. In order to rule out all inefficient stable outcomes, we need to refine the social norm employed in proposition 1. Intuitively, the kinked norm imposes an upper bound on the profitability of deviations, and was therefore useful to show that efficient profiles are stable. In order to rule out the stability of inefficient profiles, we need to impose a lower bound on the profitability of deviations. We do so by imposing a "symmetric" social norm of behavior, which essentially prescribes players to mimic the strategy adopted by a deviator.

Definition 4. (*Symmetric Social Norm*) *The Symmetric Social Norm (SSN) s is described as follows for each $i \in N$ and $x'_i \in X_i$:*

$$s_{N \setminus i}(x'_i) = (\forall j \in N \setminus i, x_j \in X_j \mid x_j = x'_i).$$

We are now ready to prove the next proposition:

Proposition 2. *Let the game G satisfy conditions **P1-P4**. Then, under the Symmetric Norm of Behavior (SSN), the set of stable strategy profile of G only contains the (symmetric) Pareto efficient profile $x^e \in X_N$.*

Proof: *See Appendix.*

Finally, a relevant question to raise is whether the behavior predicted by the model of kinked demand can in general be considered rational. About this issue, it has been proved for other purposes (see Currarini & Marini (2004)), that in all symmetric supermodular games in which strategy sets are ordered, in the event of any coalitional deviation from the efficient symmetric outcome, remaining players always play a lower strategy (under PS) and a greater strategy (under NS) than every deviating coalition. This proves that the behavior postulated by the kinked demand model is fully compatible with players' rationality whenever their actions are strategic complements (see, for instance, Bulow *et al.* (1985)) and players' best response are positively sloped. The same cannot be said when games are submodular, i.e. players' actions are strategic substitutes, and their best response are negatively sloped.

4. Concluding Remarks

In this paper we have shown that, for all symmetric and monotone strategic form games, the behavior postulated by the classical model of kinked demand possesses strong stability properties. Such a result holds even stronger when players expect a fully symmetric norm of behavior by all remaining players in the event of an individual deviation. In this case, the perfectly cooperative (collusive) outcome becomes the only stable outcome of the game. As a consequence, firms may implicitly adopt such norms of behavior in order to implement tacit collusion and sustain perfectly collusive outcome in imperfectly competitive markets.

Appendix:

Proposition 1: *Let conditions P1-P2 hold on G . Then, under the Kinked Social Norm of behavior (KSN), all symmetric Pareto efficient (SPE) strategy profiles $x^e \in X_N$ are stable.*

Proof: We know by definition 1 that KSN implies $x_j \neq x'_i$ for all $x_j \in \mathbf{k}_{N \setminus i}(x'_i)$ under positive spillovers (PS) and $x_j \equiv x'_i$ for all $x_j \in \mathbf{k}_{N \setminus i}(x'_i)$ under negative spillovers (NS). Assume first positive spillovers (PS) on G and suppose that the symmetric efficient profile (PE) $x^e \in X_N$ is not stable and there exists a $i \in N$ and a $x'_i \in X_i$ such that

$$u_i(x'_i, \mathbf{k}_{N \setminus i}(x'_i)) > u_i(x^e).$$

Using PS and the fact that $\mathbf{k}_j(x'_i) \neq x'_i$ for every $j \in N \setminus i$, we obtain

$$u_i(x'_i, \dots, x'_i) \geq u_i(x'_i, \mathbf{k}_{N \setminus i}(x'_i)) > u_i(x^e)$$

and therefore, by P1,

$$\sum_{i \in N} u_i((x'_i, \dots, x'_i)) > \sum_{i \in N} u_i(x^e),$$

which contradicts the efficiency of x^e .

Assume now that under negative spillovers (NS) there exists a player $i \in N$ with a $x'_i \in X_i$ such that

$$u_i(x'_i, \mathbf{k}_{N \setminus i}(x'_i)) > u_i(x^e).$$

By NS and the fact that $\mathbf{k}_j(x'_i) \equiv x'_i$ it must be that

$$u_i(x'_i, \dots, x'_i) \geq u_i(x'_i, \mathbf{k}_{N \setminus i}(x'_i)) > u_i(x^e)$$

which, again, leads to a contradiction.

Lemma 1: *Let the game G satisfy conditions **P1-P4**. Then, there is a unique strategy profile $x^e = \arg \max_{x \in X_N} \sum_{i \in N} u_i(x)$ and it is such that, $x_1^e = x_2^e = \dots = x_n^e$.*

Proof: Compactness of each X_i implies compactness of X_N . Continuity of each player's payoff $u_i(x)$ on x implies the continuity of the social payoff function $u_N = \sum_{i \in N} u_i(x)$. Existence of an efficient profile (PE) $x^e \in X_N$ directly follows from Weiestrass theorem. We first prove that a PE strategy profile is symmetric.

Suppose $x_i^e \neq x_j^e$ for some $i, j \in N$. By symmetry we can derive from x^e a new vector x' by permuting the strategies of players i and j such that

$$\sum_{i \in N} u_i(x') = \sum_{i \in N} u_i(x^e)$$

and hence, by the strict quasiconcavity of all $u_i(x)$, for all $\lambda \in (0,1)$ we have that:

$$\sum_{i \in N} u_i(\lambda x' + (1 - \lambda)x^e) > \sum_{i \in N} u_i(x^e).$$

Since, by the convexity of X , the strategy vector $(\lambda x' + (1 - \lambda)x^e) \in X_N$, we obtain a contradiction. Finally, by the strict quasiconcavity of both individual and social payoffs in each player's strategy, the efficient profile x^e can be easily proved to be unique.

Proposition 2: *Let the game G satisfy conditions P1-P4. Then, under the Symmetric Norm of Behaviour (SSN), the set of stable strategy profile of G only contains the (symmetric) Pareto efficient profile $x^e \in X_N$.*

Proof: Consider first the efficient profile x^e , which, by Lemma 1, must be symmetric. Suppose player i has a profitable deviation x'_i . Using the Symmetric Social Norm (SSN), the expected payoff for i would be $u_i(x'_i, \dots, x'_i)$. By symmetry, this same payoff level would be obtained by all other players in $N \setminus i$. We conclude that

$$\sum_N u_k(x'_i, \dots, x'_i) > \sum_N u_h(x^e)$$

which contradicts the efficiency of x^e . We next show that all inefficient profiles are not stable. The argument for inefficient symmetric profiles is trivial: thanks to the Symmetric Social Norm (SSN), it is enough for any player i to switch to the efficient profile to improve upon any inefficient strategy profile. Consider then an asymmetric profile x' . Let i be one player such that $u_i(x') < u_i(x^e)$ (obviously, such a player must exist by efficiency of x^e and inefficiency of x'). By continuity of payoffs, there exists some strategy \bar{x}_i close enough to x^e_i such that

$$u_i(x^e) - u_i(\bar{x}_i, \dots, \bar{x}_i) < u_i(x^e) - u_i(x').$$

Since the profile $(\bar{x}_i, \dots, \bar{x}_i)$ can be induced by player i thanks to SSN, player i has a profitable deviation, and the result follows.

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